Motion Planning using Hierarchical Graphs and Localized Tessellation of Two-Dimensional Space

Simon Pirsch

18.11.2021

Final: Bachelor Informatik - Games Engineering

Supervisor: Prof. Gudrun Klinker, Ph.D.

Advisor: Daniel Dyrda, M.Sc

Motivation – Video Games

- Days Gone: https://www.youtube.com/watch?v=bGej8K1r8KI

https://blog.playstation.com/2019/04/25/fighting-the-overwhelming-hordes-of-days-gone/

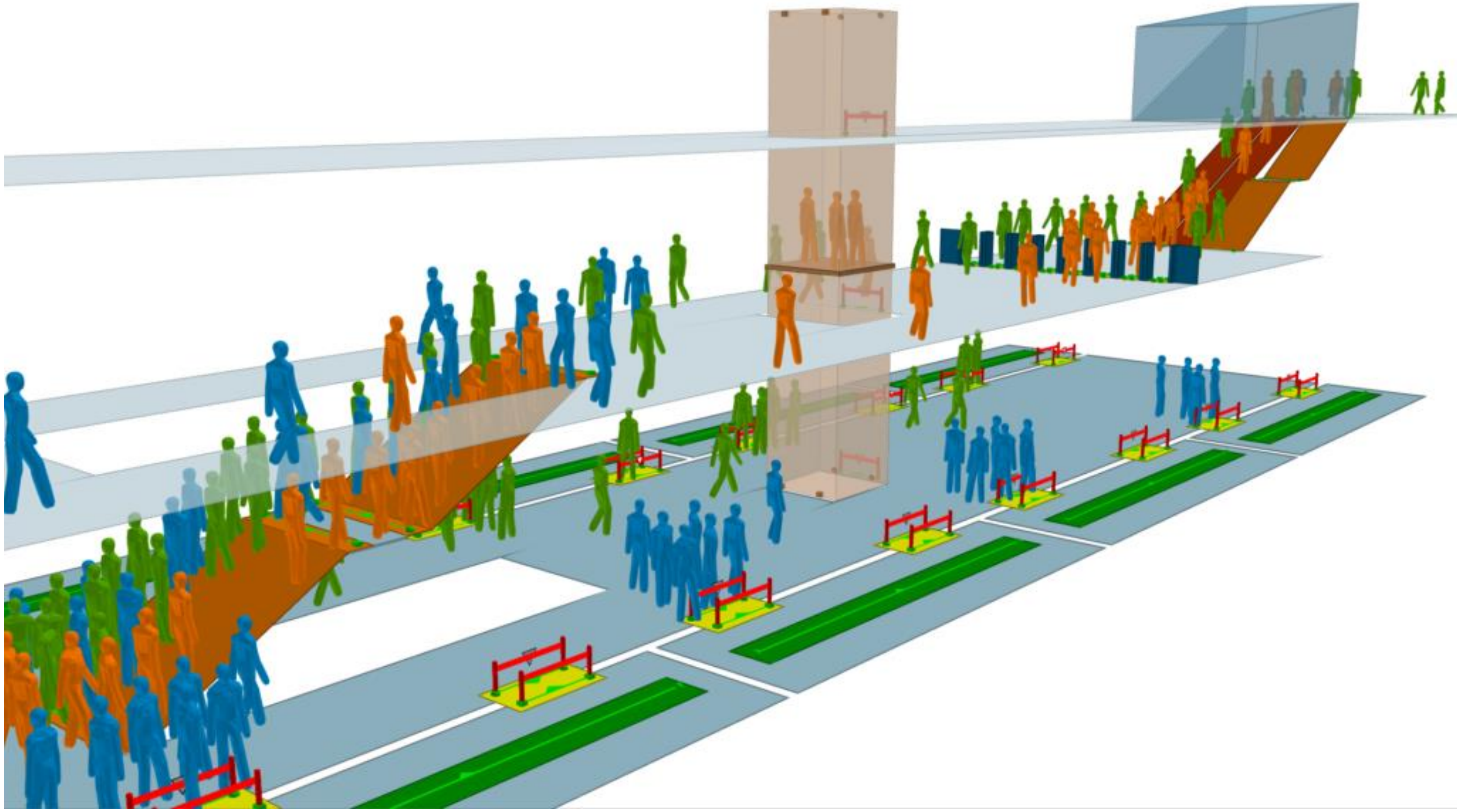
Motivation – Real-Time Strategy

- Age of Darkness: https://www.youtube.com/watch?v=rc3PzCoUd0Y

https://www.pcgamesn.com/age-of-darkness-final-stand/steam-early-access

Motivation – Crowd Simulation

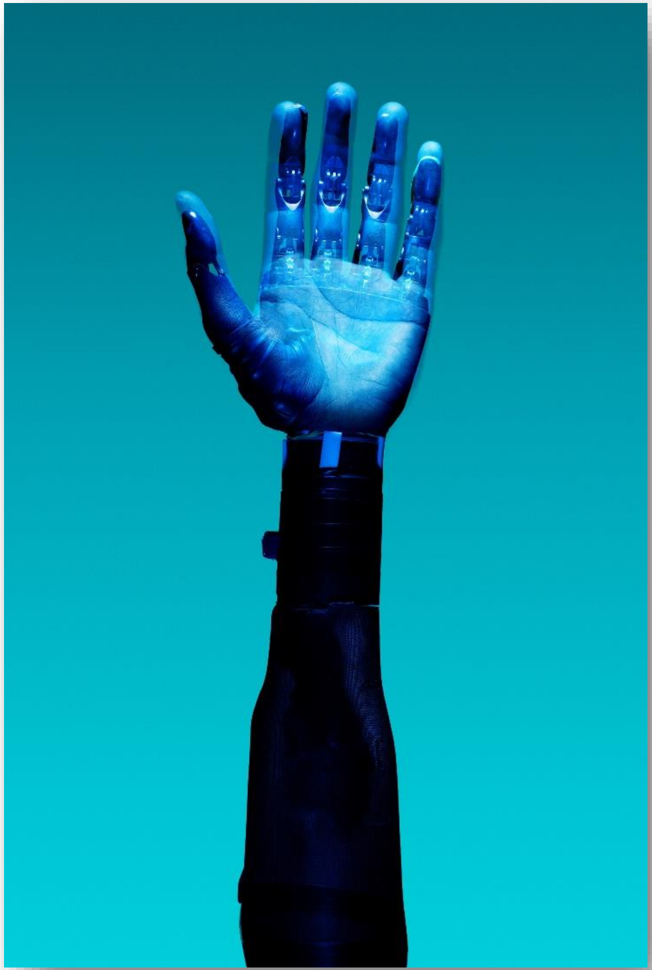
- MassMotion: https://www.oasys-software.com/products/pedestrian-simulation/massmotion/



https://www.oasys-software.com/news/launch-of-massmotion-multi-language/

Introduction

- **Motion Planning**
- **Hierarchical Graphs**
- **Localized Tessellation 2D**



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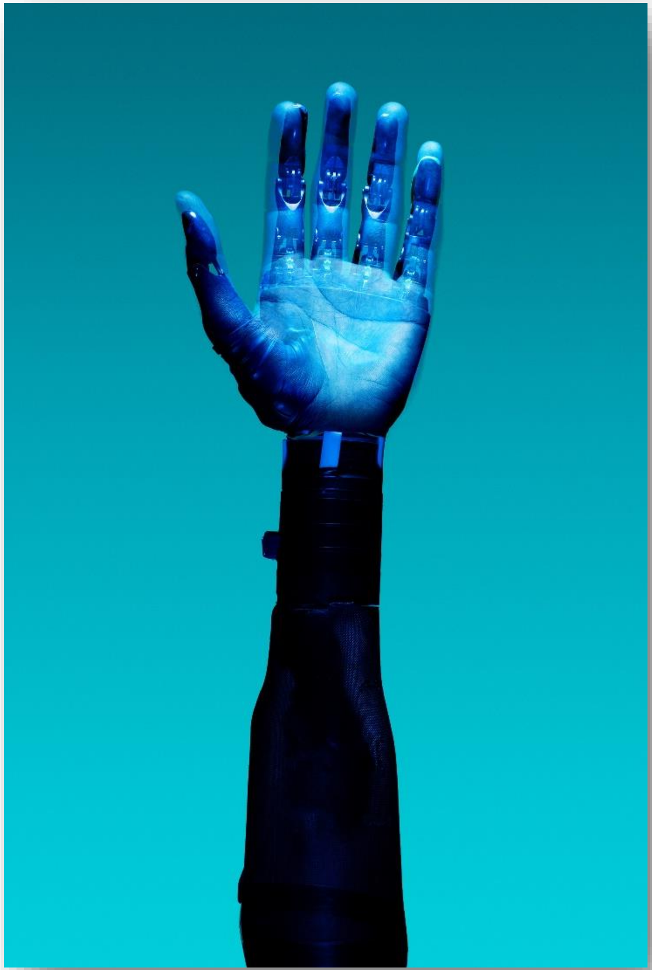

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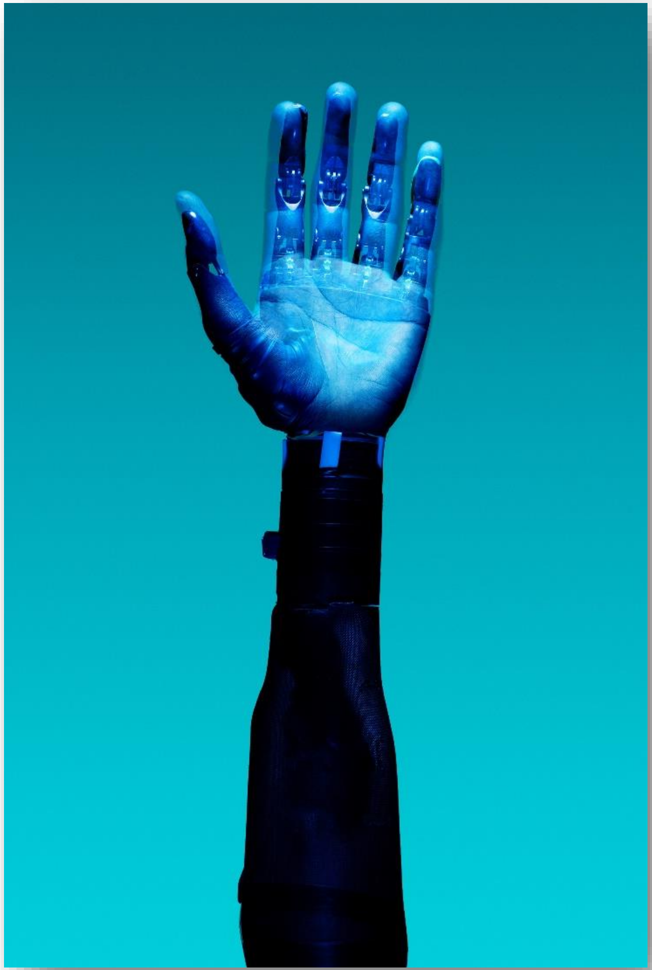

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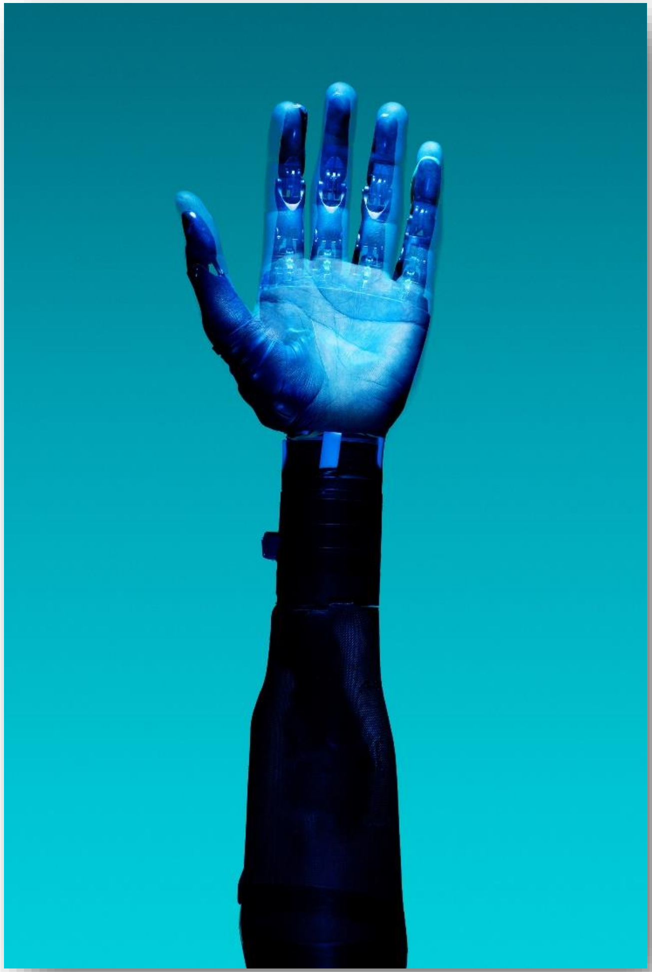
- **Localized Tessellation 2D**
  - 3D Surfaces
  - Topological aware tessellation
  - Reduce search graph complexity



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- Optimality and Performance:
  → Introduce path sub-optimality to improve performance

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→ Scalability & Performance

Related Work

- Artificial Intelligence: “Solving Problems by Searching”

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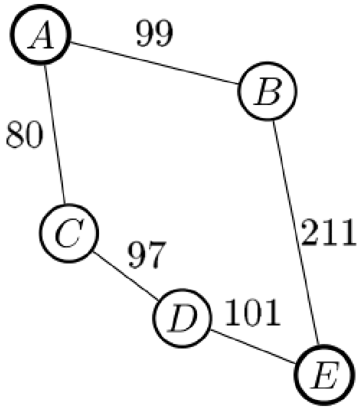
  - **Breadth-First Search**:
    - Frontier: FIFO Queue
    - Explored Set



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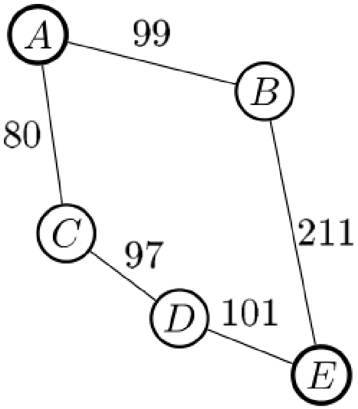

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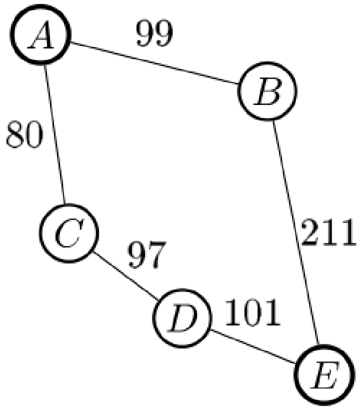
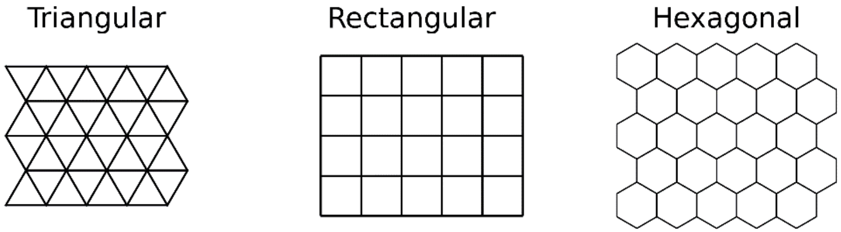
  - **A***:
    - Heuristic h: "$euclidean\ distance\ to\ goal\ node$"
    - Ordering $f = g + h$
    - Complexity: $\mathcal{O}\big((|E| + |V|)\log|V|\big)$



Regular Tessellation: Grid

- Simple regular polygon:
  - No holes or self-intersection
  - Equilateral + Equiangular



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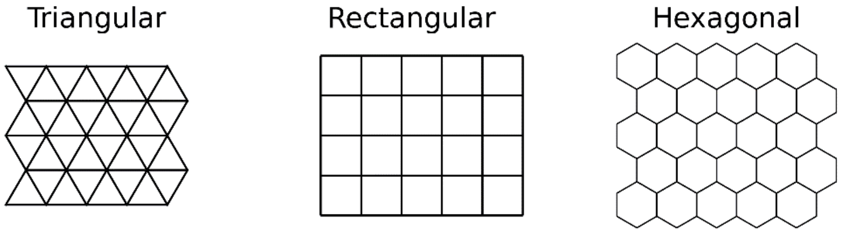
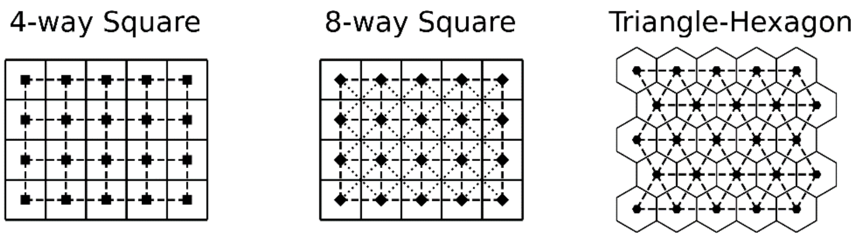
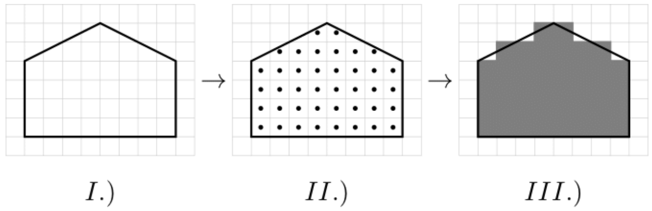


+ Simple storage, Uniformity

- Connectivity:
  - Shared edges: N_e
  - Shared vertices: N_v
  - Graph duality

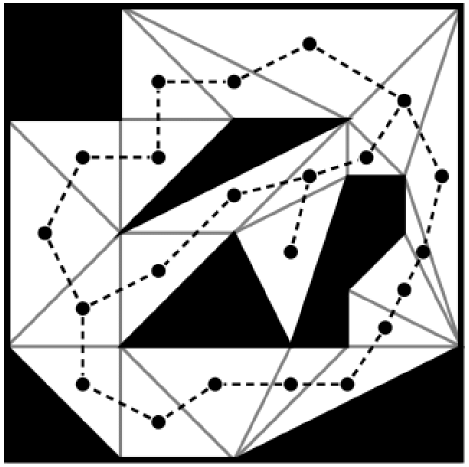


– Rasterization, Accuracy



Irregular Tessellation: NavMesh

- Constrained Delaunay Triangulation (CDT):
  - Obstacle dense area → more triangles
  - Represent arbitrary complex polygons
  - Maximize minimal internal angle
  - Produces good search graph
  - Shared unconstrained edges: N_{ue}



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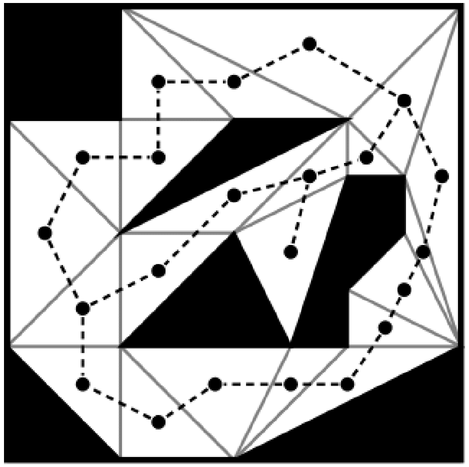
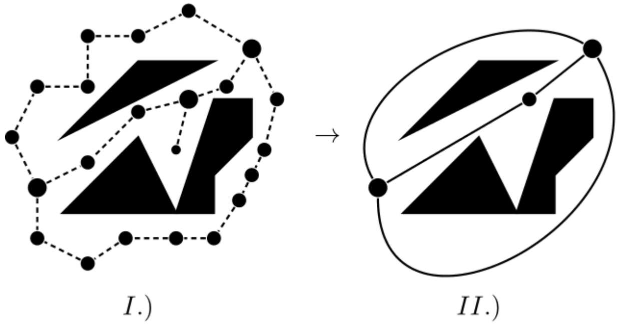


- Search (multi-)graph reduction:

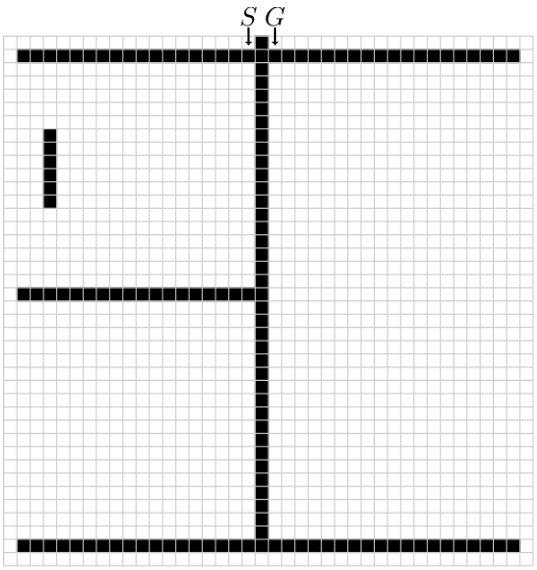


Hierarchical Graphs – HPA*

- Hierarchical Path-Finding A*

- Square grid based

- Example:
  - 40×40 maze

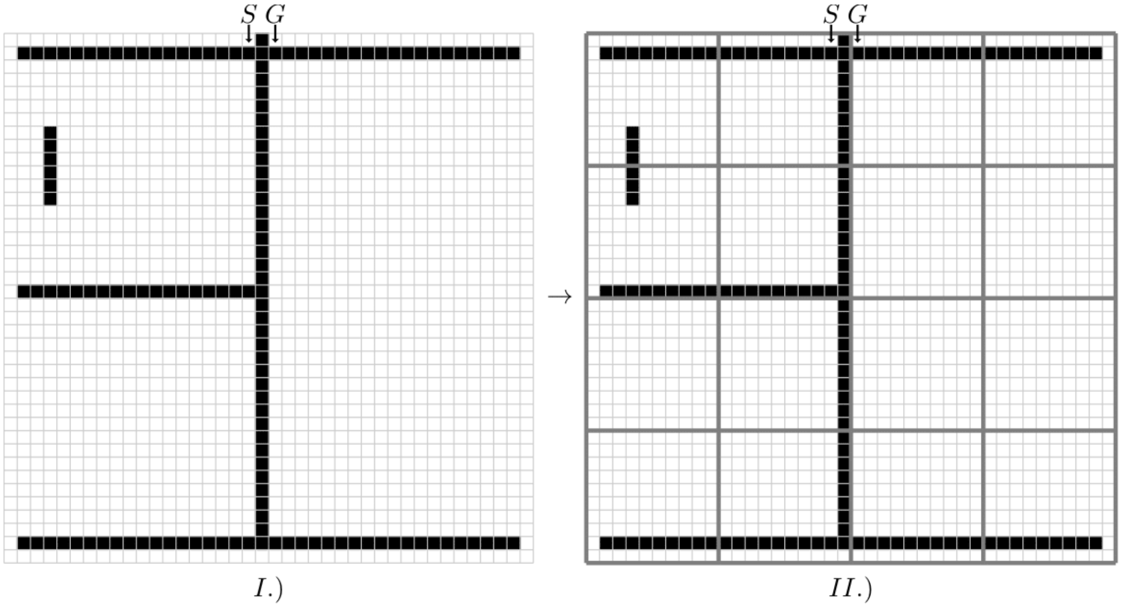


$I.)$

- Initial maze: $I.)$

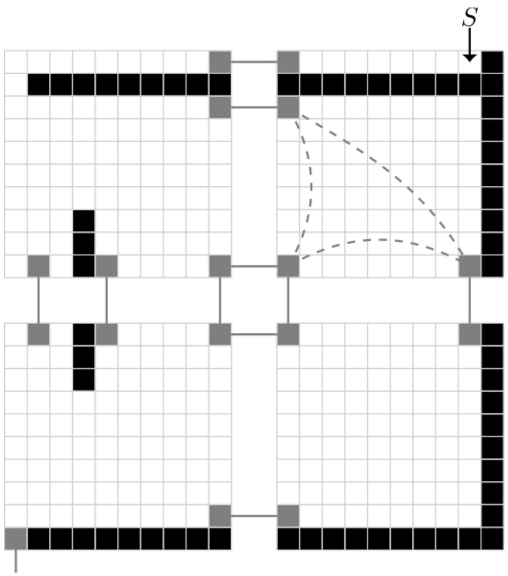
Hierarchical Graphs – HPA*

- Hierarchical Path-Finding A*
- Square grid based
- Example:
  - 40×40 maze
  - 16 10×10 clusters
- Initial maze: $I.)$
- Global level of clusters: $II.)$



HPA*: Linked Local Clusters

Inter-edges:



Constant cost: 1

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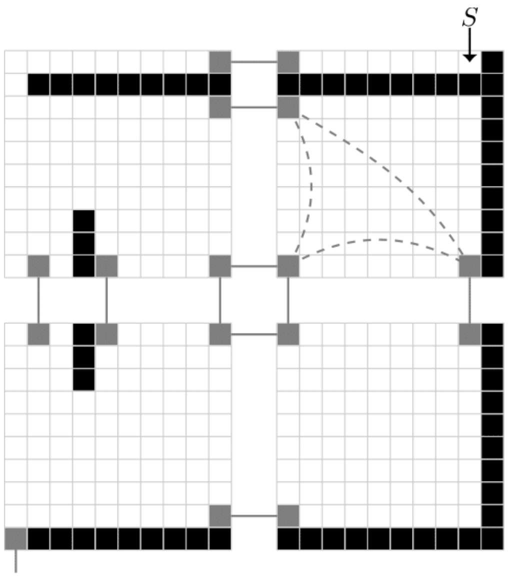

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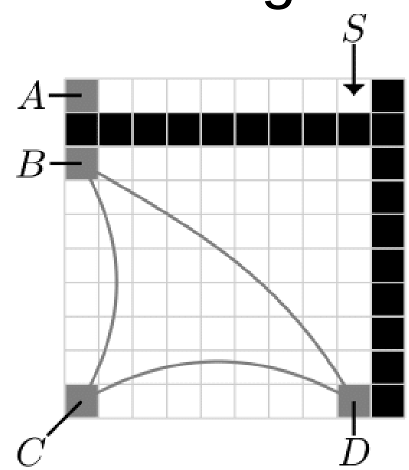
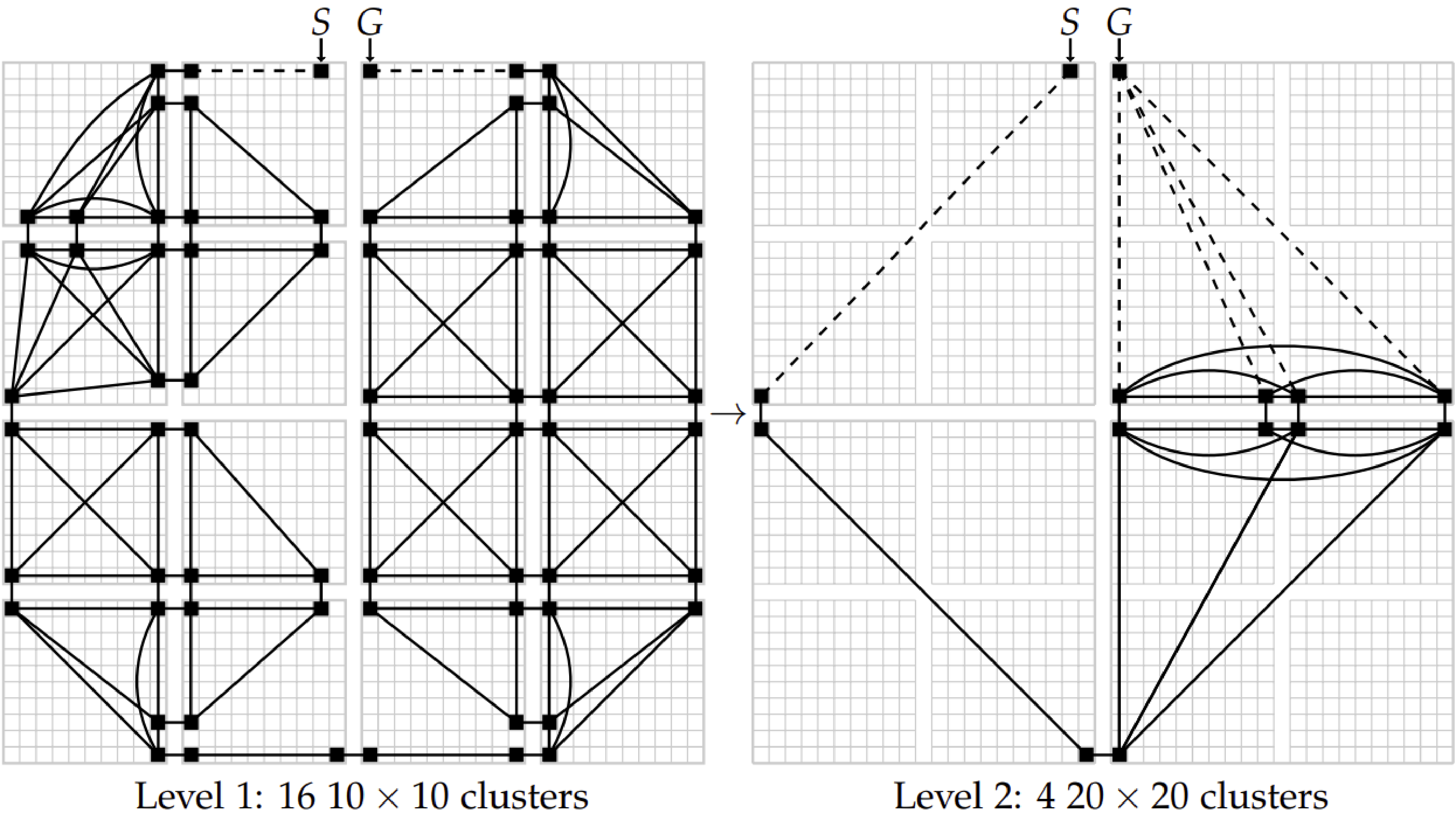
Intra-edges:



Calculated costs:

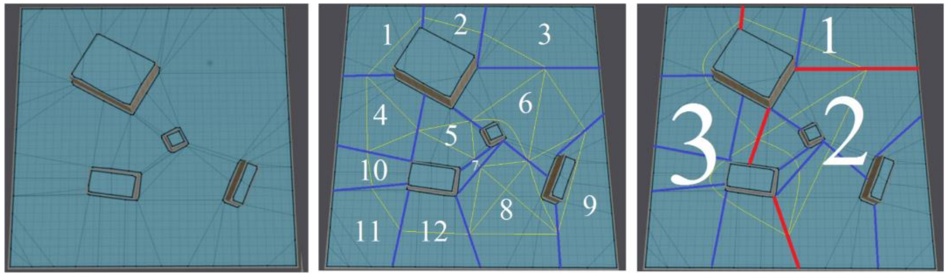
	A	*B*	*C*	*D*
A		∞	∞	∞
B			7.00	10.94
C				8.00
D				

HPA*: Abstraction Levels



Hierarchical Graphs – HNA*

- Hierarchical NavMesh Path-finding algorithm



- NavMesh based
  - Convex polygons
  - Initial graph $G_0 = (V_0, E_0)$
  - Recursively partition η nodes $G_i = (V_i, E_i)$
  - Upper limit $0 < i \leq m \in \{\lceil \log_\eta |V_0| \rceil, L_{max}\}$

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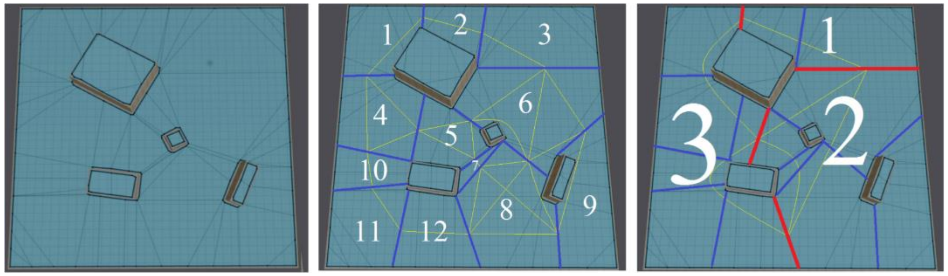

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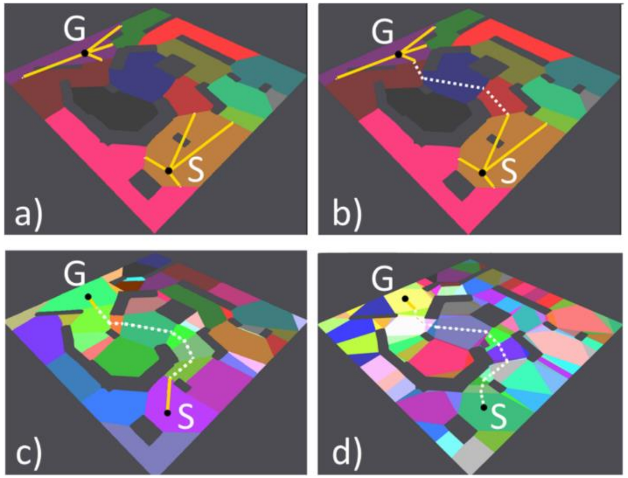
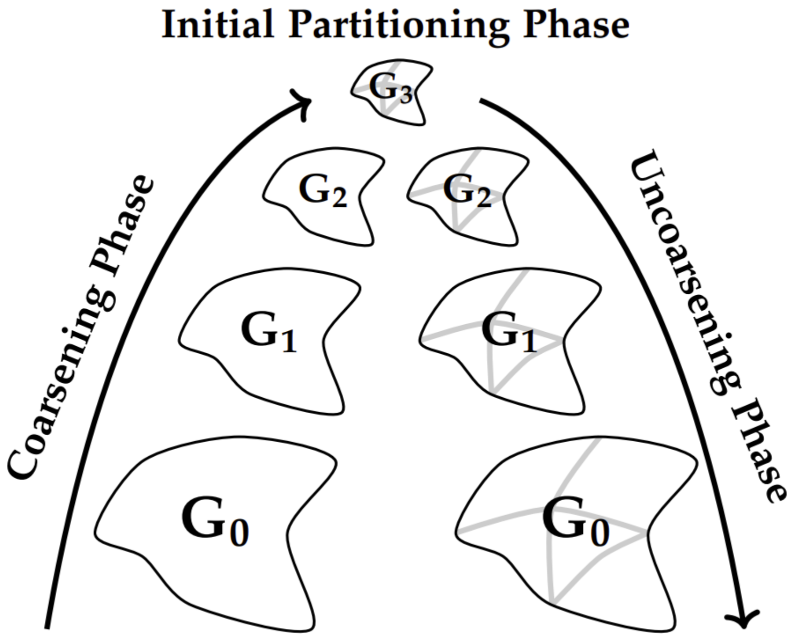


- MultiLevel k-way Partitioning (MLkP)
  - Balanced amount of components
  - Reduce edges between components
  - METIS

HNA*: MLkP Phases

- Coarsening Phase:

  recursively collapse nodes into smaller graphs G_{i+1}



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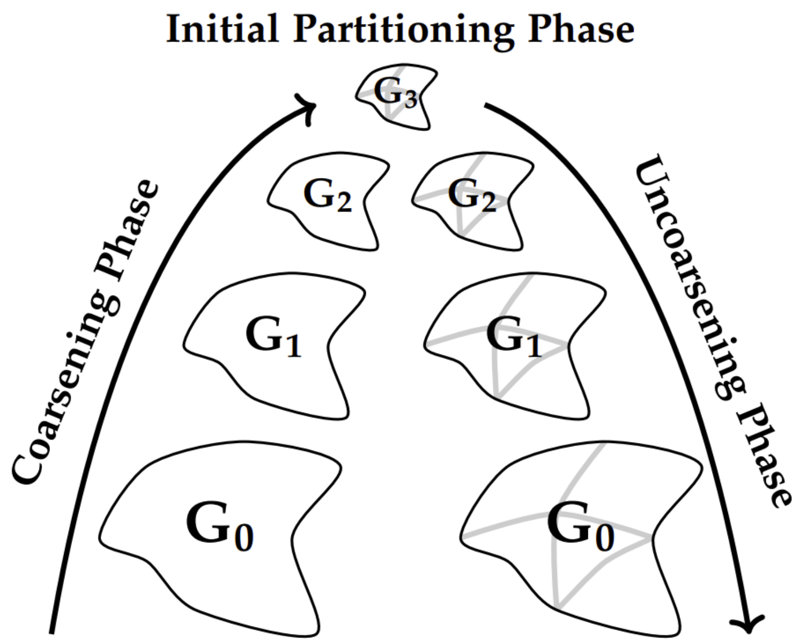
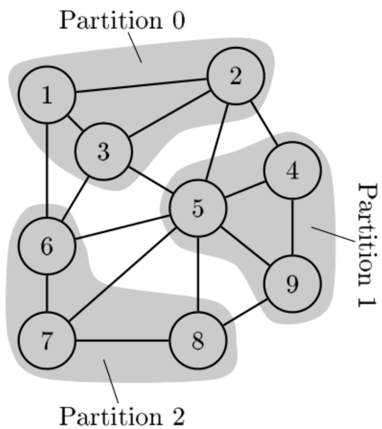


- Initial Partitioning Phase:



$$w_v: V \to \mathbb{N}$$

$$w_v(v_i) = 1 \Leftrightarrow v_i \in V_0$$

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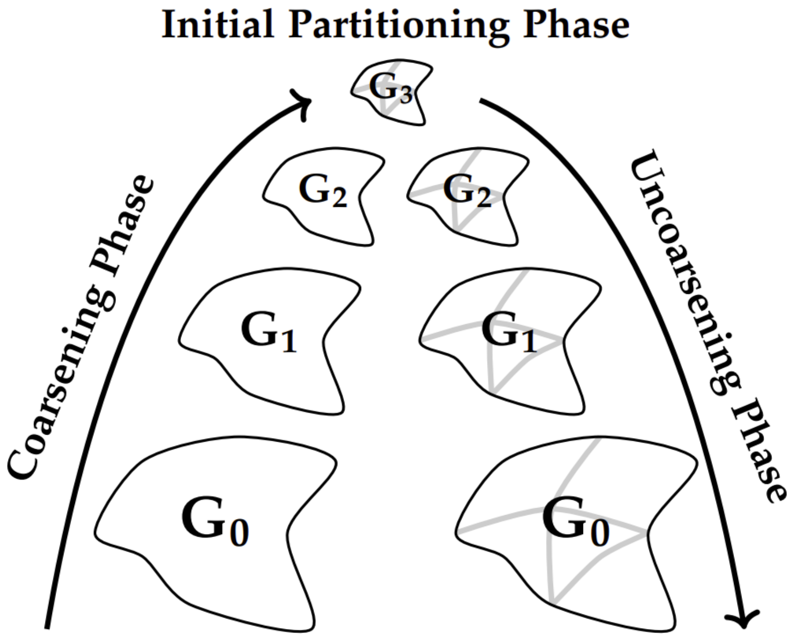

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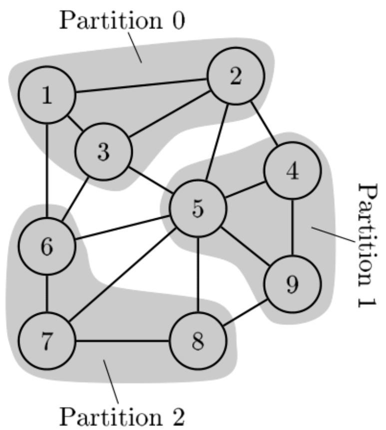

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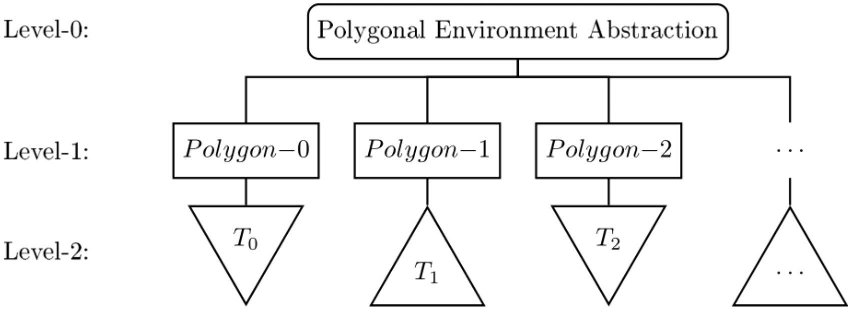
- Uncoarsening Phase:
  iteratively refine the partitioning scheme

MPHGLT2D – Introduction



- Depth = Level of detail
- Level-0:
- Level-$i \in (0..n)$:
- Level-$n = 2$:

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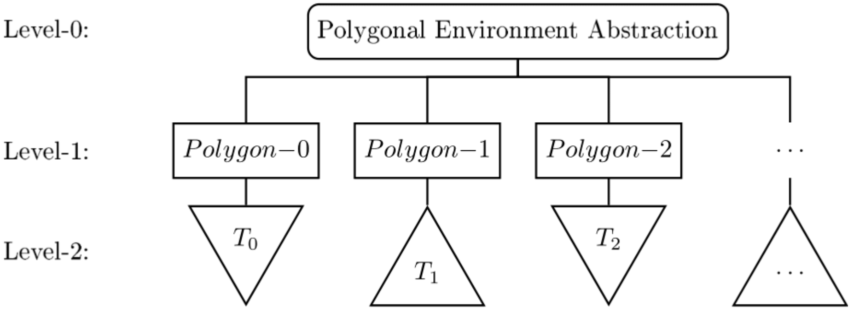

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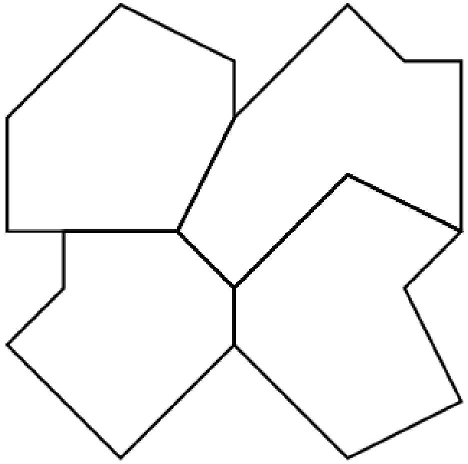

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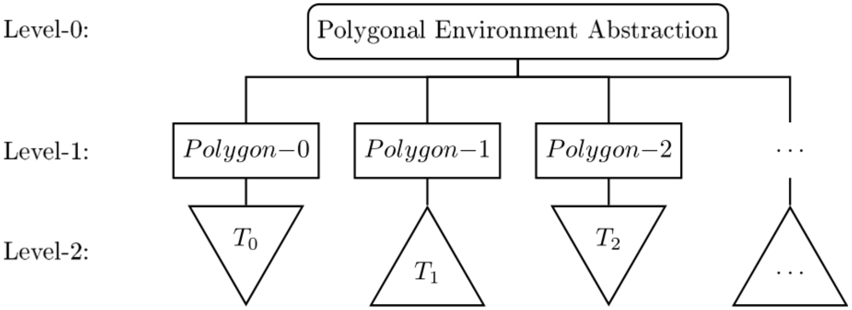

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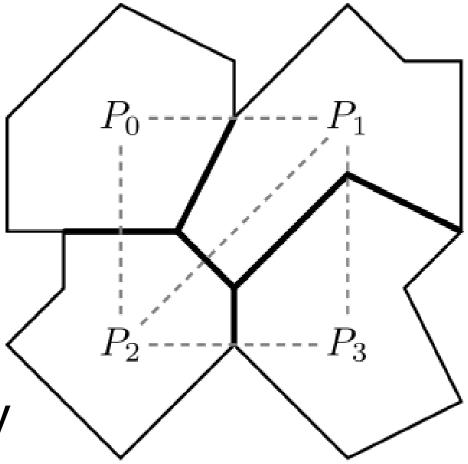

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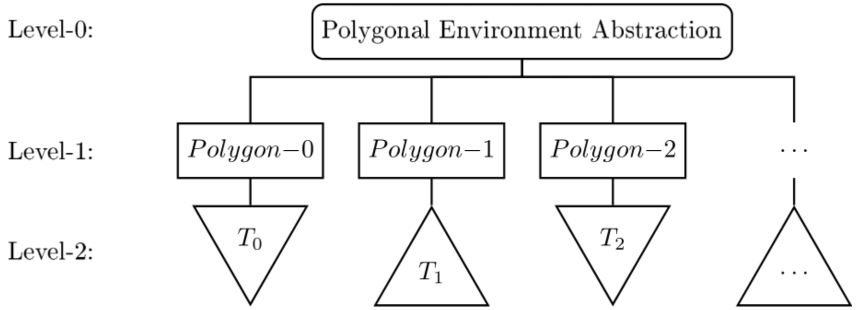

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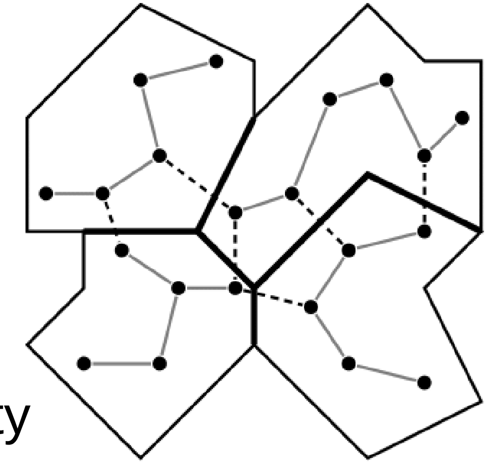
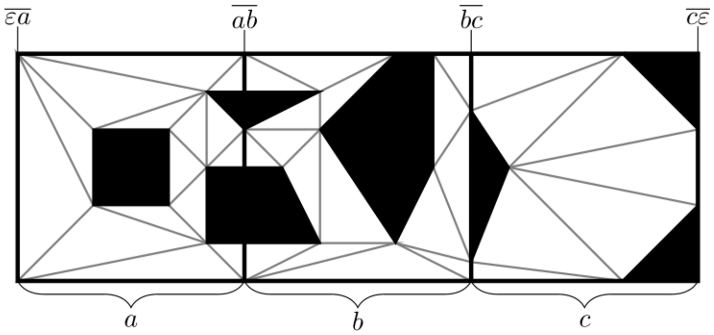
  - Parameter n: highest level of detail
  - Triangulation level (CDT)



MPHGLT2D – Example

- Example environment (CDT)
  - Square Regions: a, b, c (3×3)
  - Line Borders: $\overline{\varepsilon a} = \overline{a\varepsilon}$ (3)
  - 3 Levels: $[0..2]$



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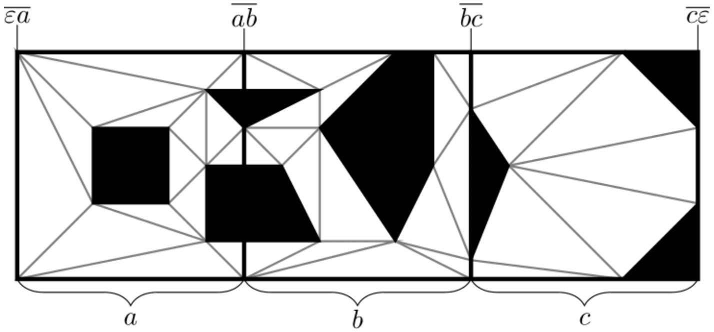
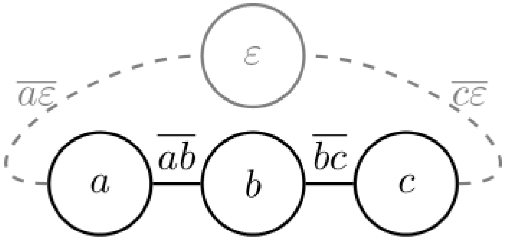


- Search graphs:
  - Level-1 graph



How to search?
Values of $\overline{ab}, \overline{bc}$?

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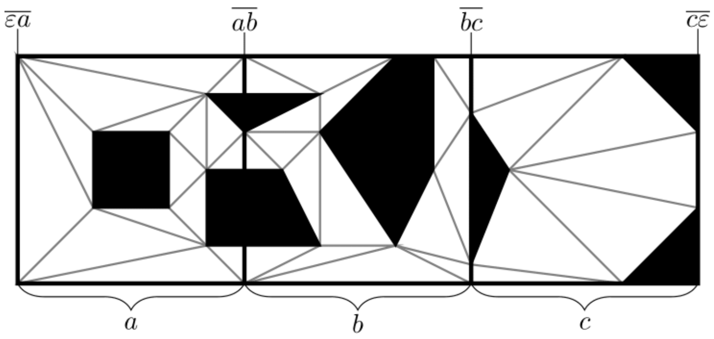

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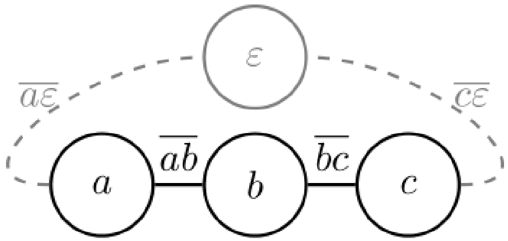
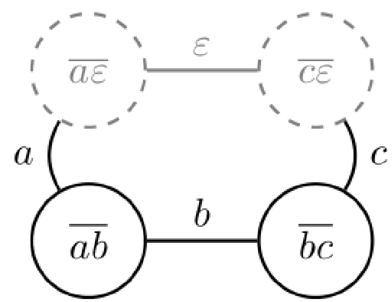


How to search?
Values of $\overline{ab}, \overline{bc}$?

  - Level-1 line graph



Convert:
Vertex ⟷ Edge

MPHGLT2D – Search

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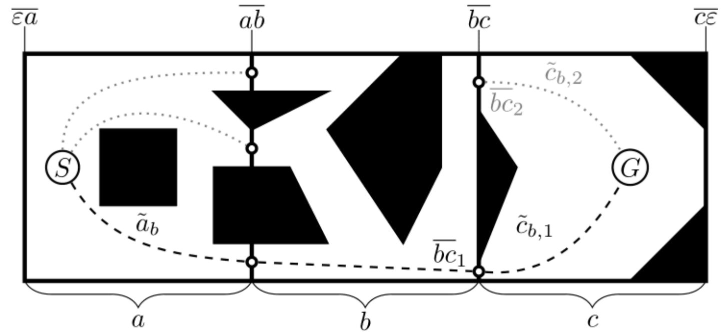
- Environment:
  - Start node S
  - Goal node G
  - Gateways $\overline{bc}_j$ of border $\overline{bc}$
  - Sub-region $\tilde{a}_b$ in region a to $\overline{ab}$



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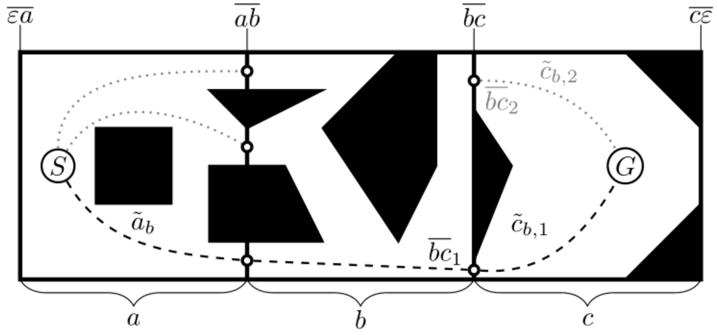

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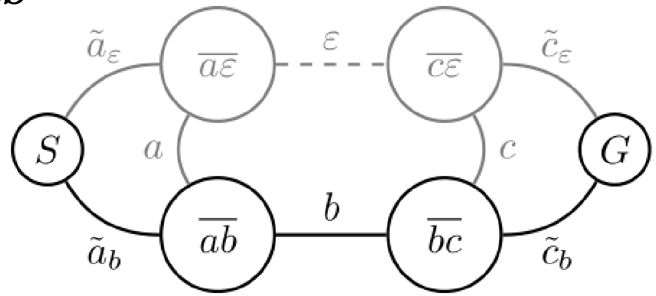

- Level-1 line graph search:
  - Edge weights?
  - Connect & calculate:
    - Start node via $\tilde{a}_{b,i}$: $distance(S, \overline{ab}_i)$
    - Gateways in b: $distance(\overline{ab}_i, \overline{bc}_j)$
    - Goal node via $\tilde{c}_{b,j}$: $distance(\overline{bc}_j, G)$

MPHGLT2D – Region Analysis

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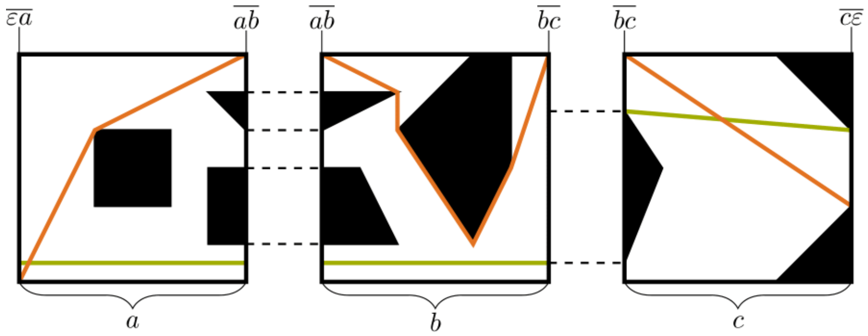
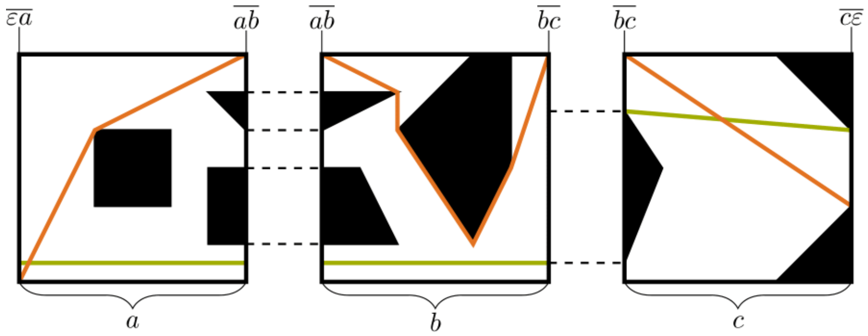
- Region $b = \overline{\overline{ab}\,\overline{bc}}$
- Monte Carlo Simulation:
  - Mean value $\overline{\chi}$ **avg**
  - Standard deviation σ **std**
  - Minimum shortest path **min**
  - Maximum shortest path **max**



MPHGLT2D – Region Analysis

+

- Region $b = \overline{\overline{ab}\;\overline{bc}}$
- Monte Carlo Simulation:
  - Mean value $\bar{\chi}$ **avg**
  - Standard deviation σ **std**
  - Minimum shortest path **min**
  - Maximum shortest path **max**



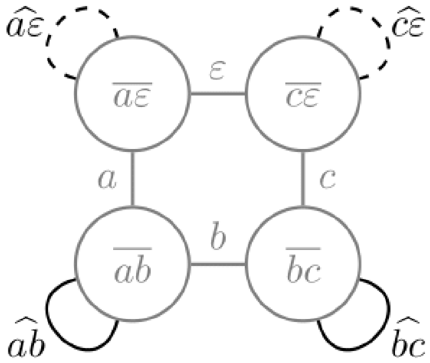
	avg	std	min	max
Square	3.23	**0.26**	3.00	4.24
Region *a*	4.28	**0.44**	3.44	5.50
Region *b*	6.71	**1.84**	3.02	9.57
Region *c*	3.80	**0.13**	3.51	4.08

Uniform distribution along borders

MPHGLT2D – Border Analysis

+

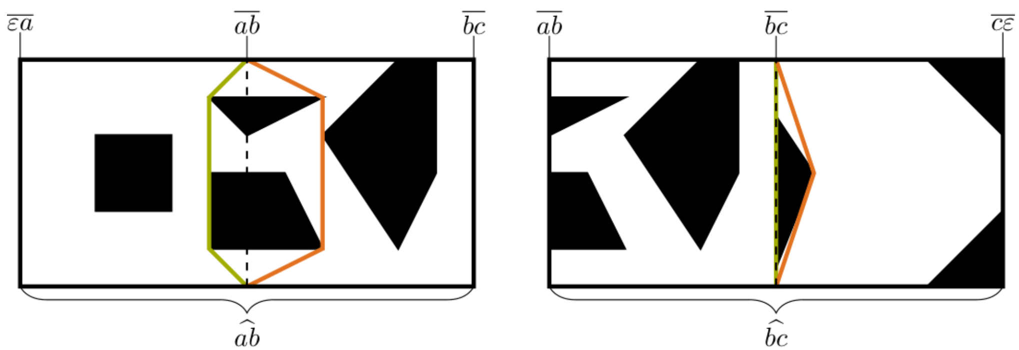
- Reflexive edges $\widehat{a\varepsilon}, \widehat{ab}, \widehat{bc}, \widehat{c\varepsilon}$

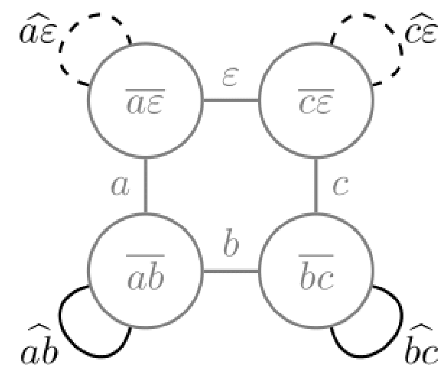


MPHGLT2D – Border Analysis

+

- Reflexive edges $\widehat{a\varepsilon}, \widehat{ab}, \widehat{bc}, \widehat{c\varepsilon}$
- True maximum shortest path
- False maximum shortest path





	avg	std	min	max
Line	1.00	**0.71**	0.00	3.00
Border $\overline{ab}$	2.35	**1.87**	0.33	5.83
Border $\overline{bc}$	1.60	**1.32**	0.33	3.60

MPHGLT2D – Use Cases

“not knowing exact paths”

- Path approximations:

- Dynamic adaptions:

MPHGLT2D – Use Cases

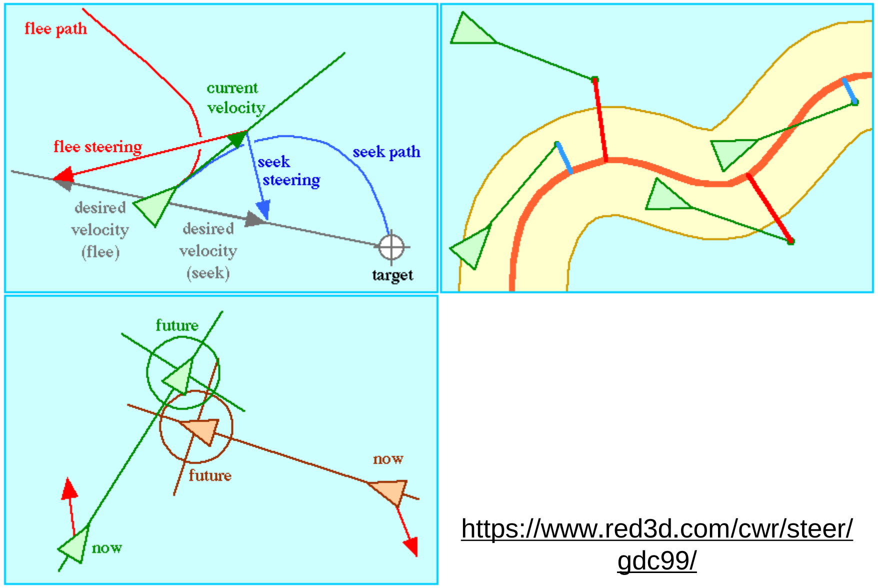
“not knowing exact paths”

- Path approximations:
  - (Pre-)calculate expected paths of all regions
  - Calculate & render exact pathing only for regions on screen
- Dynamic adaptions:

MPHGLT2D – Use Cases

"not knowing exact paths"

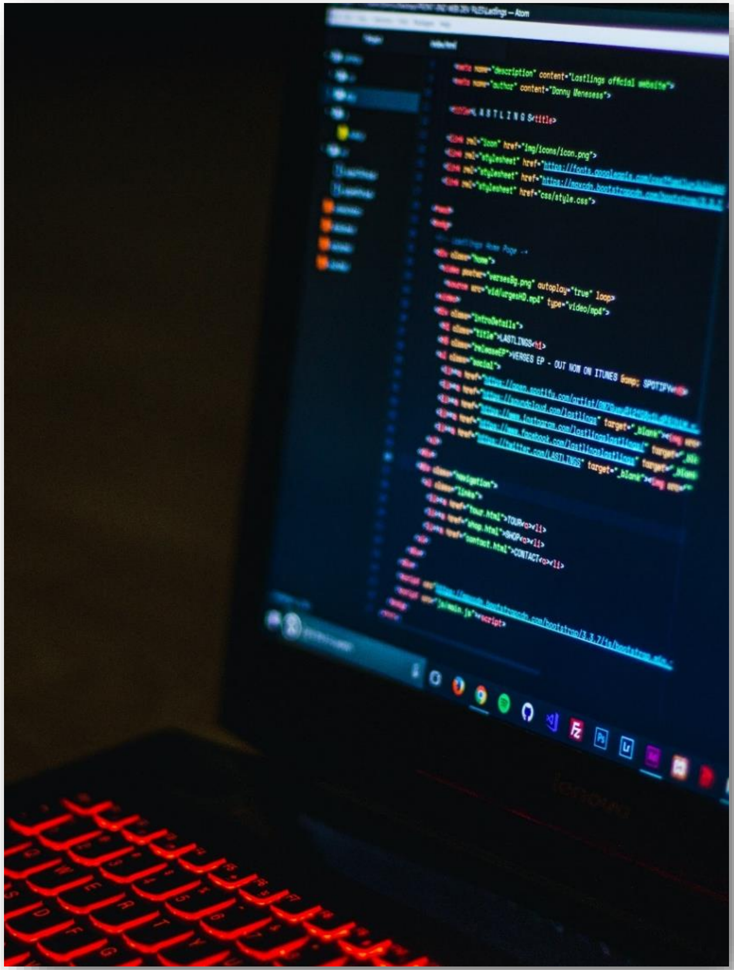
- Path approximations:
  - (Pre-)calculate expected paths of all regions
  - Calculate & render exact pathing only for regions on screen
- Dynamic adaptions:
  - Steering behaviors:
    - Path following
    - Collision avoidance
  - Congestions
  - Etc.



https://www.red3d.com/cwr/steer/gdc99/

Implementation - MPHGLT2D

- C++20
- CDT: CGAL 5.2.3
- A*: Boost 1.75
- IDE: CLion
- Compiler: GCC, clang
- Build: CMAKE 3.20+
- Visualization: Qt5
- Testing: doctest
- Package manager: msys2
- Python3



https://www.pexels.com/photo/photo-of-turned-on-laptop-computer-943096/

Conclusion

- NavMeshes:
  - – Geometrically more complex than grids
  - + Good space abstraction & CDT beneficial search graph
- HPA*:
  - – Grid based
  - + Caching performant & useful for RTS
- HNA*:
  - + NavMesh based
  - – Uses external tools, complex
- MPHGLT2D:
  - + Tries to combine aspects of both approaches
  - – Not yet fully implemented

Conclusion

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Hierarchical graphs provide good heuristics

Future Work & Discussion

- Reduce user guidance & implement all desired features
- Make full use of C++20
- Use A* variations: IDA*, TRA*, D*, etc.
- Proper path refinement & self-learning environment
- Parallelize algorithms on GPU via CUDA / OpenCL
- Integrate framework into game prototype

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I. Attribution free images from: https://www.pexels.com/

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