Technische Universität München

Institute of Flight System Dynamics

Aptitude Test Automatic Control (SAMPLE) – SOLUTION

Examiner: Prof. Dr.-Ing. Florian Holzapfel

2022-12-24

Name	
Application nr.	
Room	ONLINE INFORMATION MATERIAL
Seat	
ID Check *(to be filled by staff)*	

Task	max. Points	Points
1	7	
2	5	
3	10	
4	3	
5	7	
6	3	
7	5	
8	3	
9	7	
10	5	
11	3	
12	3	
Σ	60	

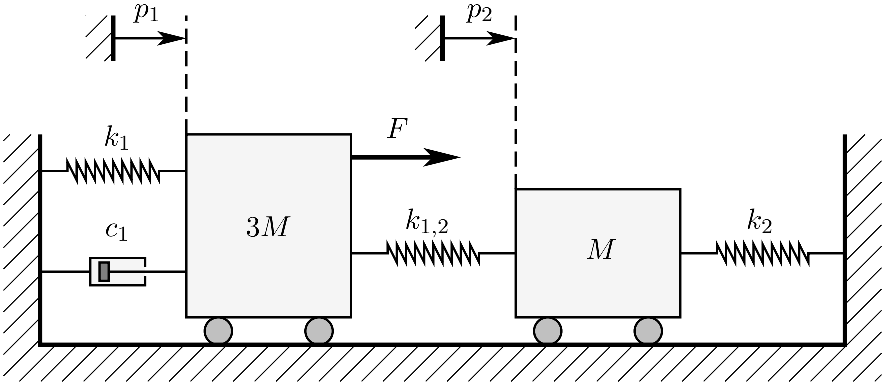
(to be filled by staff)

Exam
Editing Time: 1h

Version: unspecified

Good luck!

Task 1: **Modeling** *(ca. 7 points)*



Consider two frictionless carts with the masses $3M$ and M. The left cart is connected to the left wall by a spring with the stiffness k_1 and a damper with the damping coefficient c_1, the right cart is connected to the right wall by a spring with the stiffness k_2. The left cart is excited by an external force F. Additionally, both carts are coupled with a spring with a stiffness of $k_{1,2}$. In the equilibrium position $p_1 = p_2 = 0$ all springs are tension-free. All springs and dampers are considered massless.

a) Use Newton's second law to derive the equation of motion of the left cart in the form $\ddot{p}_1 = f(p_1, \dot{p}_1, p_2, \dot{p}_2, F)$.

(ca. 2 points)

$$3M\,\ddot{p}_1 = F - k_1\,p_1 - c_1\,\dot{p}_1 + k_{1,2}\,(p_2 - p_1)$$

$$\ddot{p}_1 = \frac{1}{3M}\left(-(k_1 + k_{1,2})\,p_1 - c_1\,\dot{p}_1 + k_{1,2}\,p_2 + F\right)$$

b) Use Newton's second law to derive the equation of motion of the right cart in the form $\ddot{p}_2 = f(p_1, \dot{p}_1, p_2, \dot{p}_2, F)$.

(ca. 2 points)

$$M\,\ddot{p}_2 = k_{1,2}\,(p_1 - p_2) - k_2\,p_2$$

$$\ddot{p}_2 = \frac{1}{M}\,(k_{1,2}\,p_1 - (k_2 + k_{1,2})\,p_2)$$

c) Derive a state space model of the form $\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{b}\,u, \quad y = \mathbf{c}^{\mathsf{T}}\mathbf{x}$. The input u is the external force F and the distance $p_2 - p_1$ shall be considered as output signal y. Use the state vector $\mathbf{x} = [p_1, \dot{p}_1, p_2, \dot{p}_2]^{\mathsf{T}}$.

(ca. 3 points)

$$\begin{bmatrix} \dot{p}_1 \\ \ddot{p}_1 \\ \dot{p}_2 \\ \ddot{p}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1+k_{1,2}}{3M} & -\frac{c_1}{3M} & \frac{k_{1,2}}{3M} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_{1,2}}{M} & 0 & -\frac{k_2+k_{1,2}}{M} & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ \dot{p}_1 \\ p_2 \\ \dot{p}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{3M} \\ 0 \\ 0 \end{bmatrix} F$$

$$y = p_2 - p_1 = \begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_1 & \dot{p}_1 & p_2 & \dot{p}_2 \end{bmatrix}^{\mathsf{T}}$$

Task 2: **Laplace Transform** *(ca. 5 points)*

The differential equation

$$\dot{y}(t) + \sigma(t) * y(t) = 2 * u(t)$$

of a system is given. The initial value is $y(0) =$. Consider the following hints regarding notation and the given Laplace correspondences.

a) Determine the step response $h(t)$ and the impulse reponse $g(t)$ of the system. *(ca. 3 points)*

$$\dot{y}(t) + \sigma(t) * y(t) = 2 * u(t)$$

$$\circ\!\!-\!\!\bullet$$

$$sY(s) - \underbrace{y(0)}_{=0} + \frac{1}{s}Y(s) = 2U(s)$$

$$\Rightarrow Y(s) = \frac{2s}{s^2 - 1}U(s)\checkmark$$

$$U(s) = \frac{1}{s} \Rightarrow Y(s) = H(s) = \frac{2}{s^2 + 1}$$

$$\bullet\!\!-\!\!\circ$$

$$h(t) = 2\sin(t)\checkmark$$

$$g(t) = \dot{h}(t) = 2\cos(t)\checkmark$$

b) The system of differential equations is now given as

$$\begin{aligned}\dot{y} &= -x + 2u \\ \dot{x} &= f(y, x, u)\end{aligned} \tag{1}$$

Compute $f(x, y, u)$ such that the system of differential equations is equivalent to the system from a).

(ca. 2 points)

$$\dot{y}(t) + x(t) = 2u(t)$$

$$\circ\!\!-\!\!\bullet$$

$$sY(s) - y(0) + \underbrace{X(s)}_{=\frac{1}{s}Y(s)} = 2U(s)$$

$$\implies sX(s) = Y(s)\checkmark$$

$$\bullet\!\!-\!\!\circ$$

$$\dot{x} = f(y, x, u) = y\checkmark$$

Task 3: **Bode Diagram** *(ca. 10 points)*

The transfer function of a dynamic system can be represented by the series connection of the two transfer functions $G_1(s)$ and $G_2(s)$:

$$G(s) = \frac{Y(s)}{U(s)} = G_1(s)\, G_2(s)$$

The following diagram contains approximations of the magnitude and phase response curves for the element $G_1(s)$. The transfer function $G_2(s)$ is:

$$G_2(s) = \frac{20s + 4000}{(s + 20)^2}$$

Tasks: see next pages.

a) Determine the corner frequencies of $G_2(s)$ as well as approximations of the initial value $|G_2(\omega_{min})|$ and gradient $|G_2(\omega_{min})|'$ of the magnitude response in dB and dB/decade and the initial value of the phase response $\angle G_2(\omega_{min})$.

(ca. 4 points)

Hint: $\omega_{min} = 10^{-1}$

Corner frequencies:

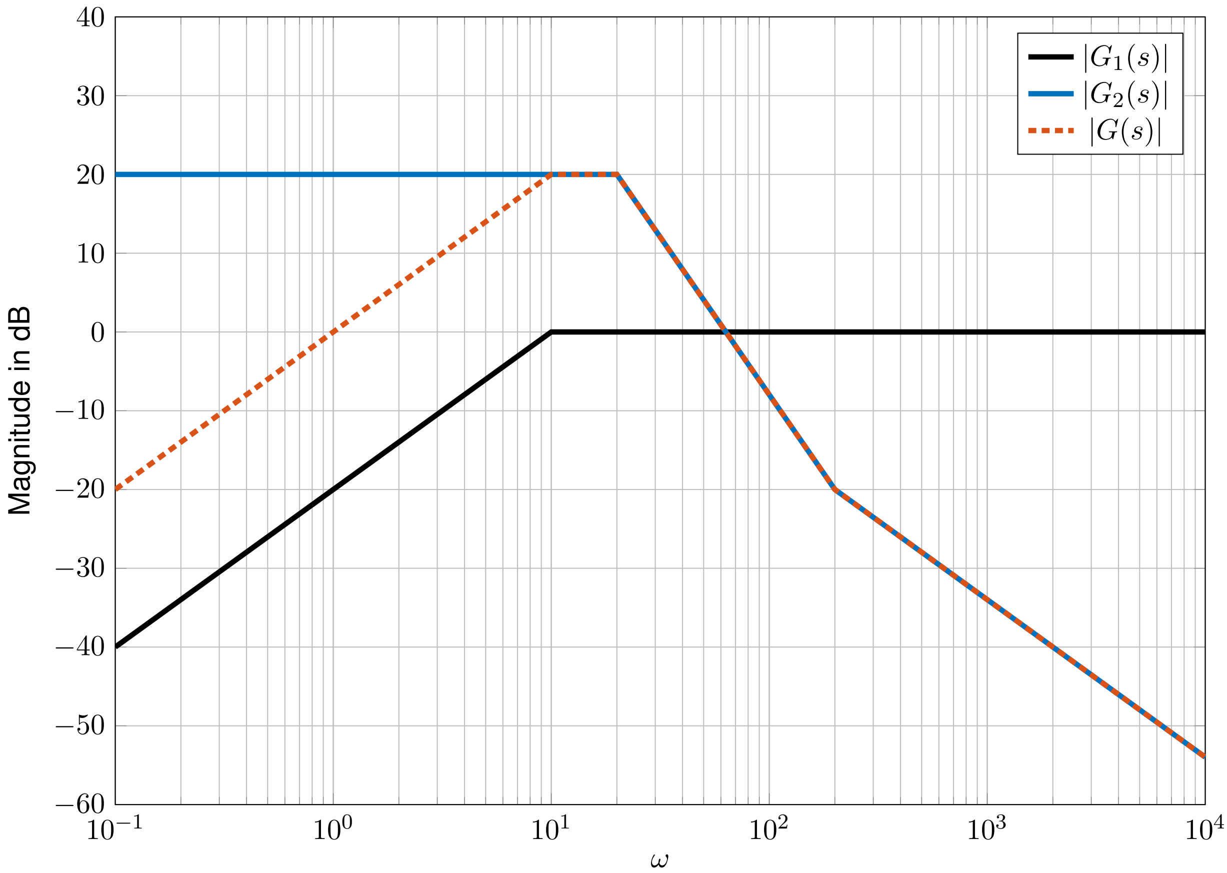
$$G_2(s) = \frac{20s + 4000}{(s+20)^2} = 10\frac{s/200 + 1}{(s/20+1)^2}$$
$$\tilde{T}_1 = \frac{1}{200} \quad \Rightarrow \quad \tilde{\omega}_1 = \frac{1}{|\tilde{T}_1|} = 200$$
$$T_{1,2} = \frac{1}{20} \quad \Rightarrow \quad \omega_{1,2} = \frac{1}{|T_{1,2}|} = 20$$

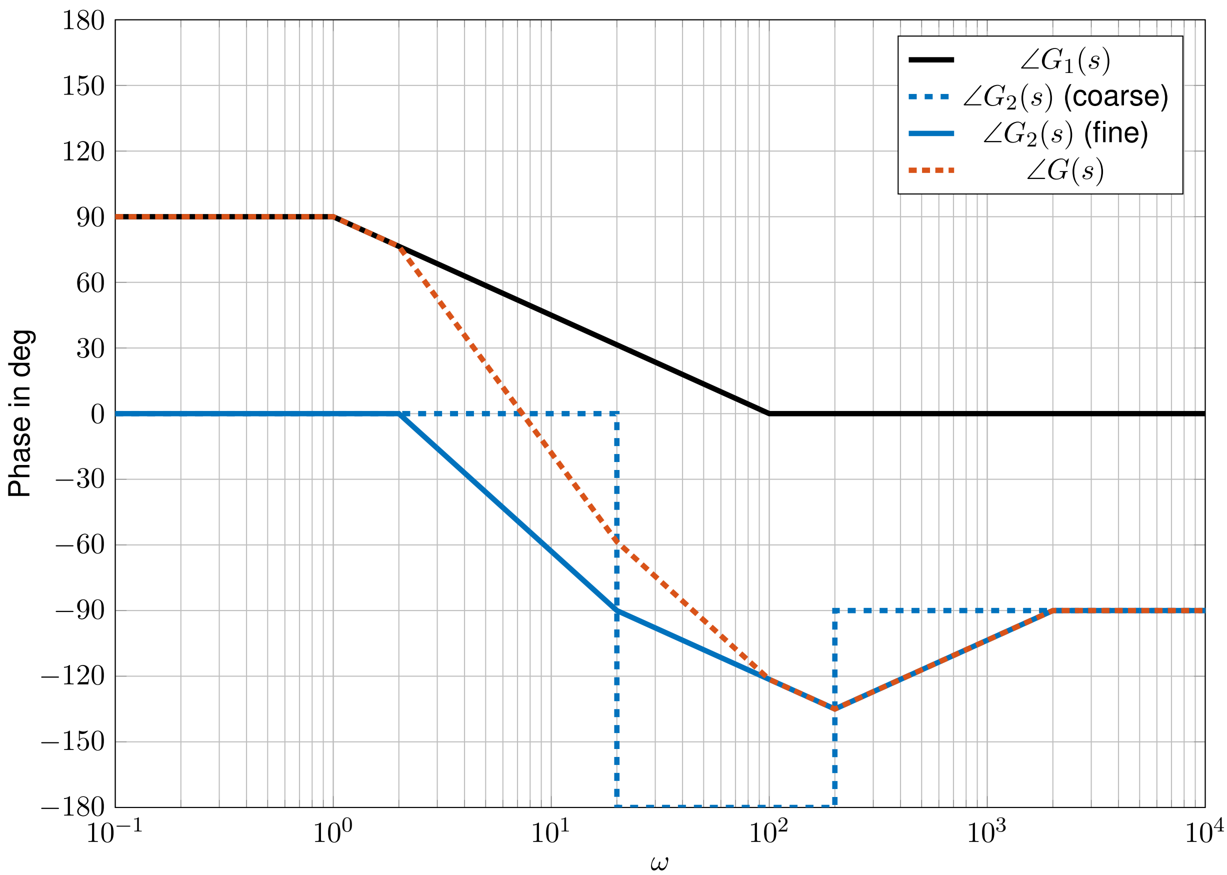
Gain $K = 10$, no poles or zeros at 0:

$$\Rightarrow \quad |G_2(\omega_{min})| \approx 20\log_{10}(10) = 20 \text{ dB}$$
$$\Rightarrow \quad |G_2(\omega_{min})|' \approx 0 \text{ dB/decade}$$
$$\Rightarrow \quad \angle G_2(\omega_{min}) \approx 0^\circ$$

b) Draw the approximation of the magnitude and phase response curves of $G_2(s)$ in the diagram on the next page. *(ca. 4 points)*

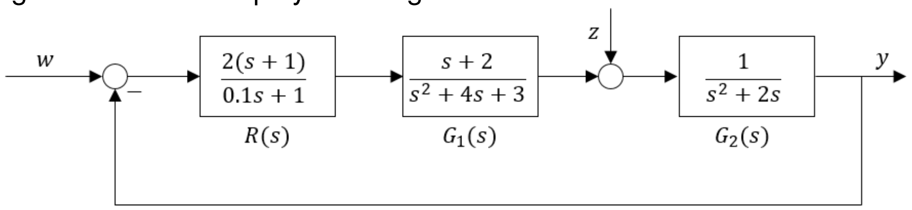
c) Finally, draw the approximation of the magnitude and phase response curves of the entire system $G(s)$ in the diagram on the next page. *(ca. 2 points)*


Magnitude in dB
ω
$|G_1(s)|$
$|G_2(s)|$
$|G(s)|$
40
30
20
10
0
−10
−20
−30
−40
−50
−60
10^{-1}
10^0
10^1
10^2
10^3
10^4


Phase in deg
ω
$\angle G_1(s)$
$\angle G_2(s)$ (coarse)
$\angle G_2(s)$ (fine)
$\angle G(s)$
180
150
120
90
60
30
0
−30
−60
−90
−120
−150
−180
10^{-1}
10^0
10^1
10^2
10^3
10^4

Task 4: System Analysis / Steady-State Accuracy *(ca. 3 points)*

The following stable closed-loop system is given:



a) Evaluate the steady-state accuracy of the closed-loop system with regard to the reference tracking behavior of the control loop (reference variable: w) assuming a step input in w and justify your statement.

(ca. 1 point)

Integration in $R\,G_1\,G_2$, closed-loop stable $\Rightarrow$ steady-state accurate reference tracking.

b) Evaluate the steady-state accuracy of the closed-loop system with regard to the disturbance behavior of the control loop (disturbance variable: z) assuming a step input in z and justify your statement.

(ca. 1 point)

No integration in $R\,G_1$ $\Rightarrow$ disturbance behavior not steady-state accurate.

c) What type of the controller $R(s)$ is used? If necessary, distinguish between ideal and real implementation.

(ca. 1 point)

Real PD controller.

Task 5: Input/Output Linearization *(ca. 7 points)*

Consider a dynamic system of the form $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u),\ y = h(\mathbf{x})$ with the states $\mathbf{x} = [x_1, x_2, x_3]^\mathsf{T}$, input signal u and output signal y:

$$\mathbf{x} \in \mathbb{R}^n \qquad \mathbf{f} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n,\ (\mathbf{x}, u) \mapsto \dot{\mathbf{x}}$$

$$u \in \mathbb{R} \qquad h : \mathbb{R}^n \to \mathbb{R},\ \mathbf{x} \mapsto y$$

a) Describe *in general terms* the principle of input-output linearization for single input single output (SISO) systems of this form.

(ca. 2 points)

The system output y is differentiated until an explicit influence by the input u appears at derivative order r (the relative degree). Solving the relation between $\overset{(r)}{y}$ and u for the input signal u yields a control law that compensates the nonlinearity of the system by inversion such that linear input-output dynamics result, with the new input $\overset{(r)}{y}$ acting as pseudo-control.

In the following the system S is considered:

$$S: \quad \mathbf{f}(\mathbf{x}, u) = \begin{bmatrix} \sin(x_2) + (x_2 + 1)\, x_3 \\ x_1^5 + x_3 \\ x_1^2 + u \end{bmatrix}, \quad h(\mathbf{x}) = x_1$$

b) Differentiate the output of the system S up to the relative degree and explicitly state the relation to the input signal.

(ca. 2 points)

$$y = x_1$$
$$\dot{y} = \dot{x}_1 = \sin(x_2) + (x_2 + 1)\, x_3$$
$$\ddot{y} = \big(\cos(x_2) + x_3\big)\, \dot{x}_2 + (x_2 + 1)\, \dot{x}_3$$
$$\ddot{y} = \big(\cos(x_2) + x_3\big)\, (x_1^5 + x_3) + (x_2 + 1)\, (x_1^2 + u)$$

c) State a pseudo-control variable ν for the system S.

(ca. 1 point)

$$\nu = \ddot{y}$$

d) Calculate a control law $u = g(\mathbf{x}, \nu)$ for the system S that linearizes the dynamics between ν and y.

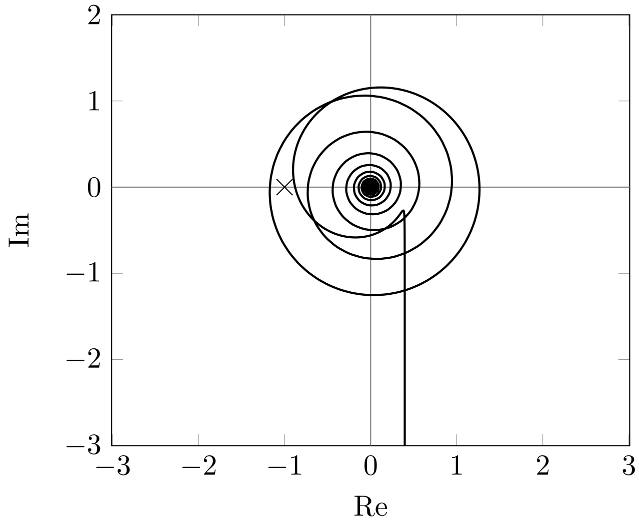
(ca. 2 points)

$$u = \frac{\nu - \big(\cos(x_2) + x_3\big)\, (x_1^5 + x_3) - (x_2 + 1)\, x_1^2}{(x_2 + 1)}$$

Task 6: **Nyquist Criterion** *(ca. 3 points)*

The following plot shows the open-loop frequency response $G(j\omega)$ of the open-loop system $G(s)$ for all relevant frequencies $\omega > 0$. The closed-loop stability shall be assessed using the general Nyquist criterion.

- Calculate the argument change W_+^* of $G(jw) + 1$ required for closed-loop stability.
- Determine the actual argument change W_+.
- Determine the stability of the closed-loop system.



$$G(s) = 220\,\exp(-s)\frac{(s+0.05)\,(s-1)}{s\,(s+10)^2\,(s-5)}$$

- one neutral and one unstable pole
  $\Rightarrow \quad W_+^* = 1\frac{\pi}{2} + 1\pi = \frac{3\pi}{2}$ (1P)
- from locus: $W_+ = -\frac{3\pi}{2}$ (1P)
- $W_+ \neq W_+^* \quad \Rightarrow$ closed-loop system unstable (1P)
- *alternative solution:*
  $\omega \in [-\infty, \infty] \quad \Rightarrow \quad W_+^* = 3\pi \neq -3\pi = W_+$

Task 7: State Observer *(ca. 5 points)*

The system Σ is given. To estimate its state $\mathbf{x}$, a state observer L is used. The estimation error $\tilde{\mathbf{x}}$ is defined as $\tilde{\mathbf{x}} := \mathbf{x} - \hat{\mathbf{x}}$.

$$\Sigma: \quad \dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} \qquad y = \mathbf{c}^\mathsf{T}\,\mathbf{x}$$

$$L: \quad \dot{\hat{\mathbf{x}}} = \mathbf{A}\,\hat{\mathbf{x}} - \mathbf{k}\,(y - \hat{y}) \qquad \hat{y} = \mathbf{c}^\mathsf{T}\,\mathbf{x}$$

a) Derive the differential equation $\dot{\tilde{\mathbf{x}}} = \tilde{\mathbf{A}}\,\tilde{\mathbf{x}}$ for the estimation error dynamics. *(ca. 2 points)*

$$\tilde{\mathbf{x}} := \mathbf{x} - \hat{\mathbf{x}} \quad \Rightarrow \quad \dot{\tilde{\mathbf{x}}} = \dot{\mathbf{x}} - \dot{\hat{\mathbf{x}}} = \mathbf{A}\,(\mathbf{x} - \hat{\mathbf{x}}) + \mathbf{k}\,\mathbf{c}^\mathsf{T}\,(\mathbf{x} - \hat{\mathbf{x}}) = \underbrace{\left(\mathbf{A} + \mathbf{k}\,\mathbf{c}^\mathsf{T}\right)}_{\tilde{\mathbf{A}}}\tilde{\mathbf{x}}$$

Now the following values are given:

$$\mathbf{A} = \begin{bmatrix} -1 & 4 \\ 0 & -2 \end{bmatrix}, \qquad \mathbf{k} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \qquad \mathbf{c}^\mathsf{T} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

b) Calculate the matrix $\tilde{\mathbf{A}}$ and its eigenvalues.

(ca. 2 points)

$$\begin{aligned}
\tilde{\mathbf{A}} &= (\mathbf{A} + \mathbf{k}\,\mathbf{c}^\mathsf{T}) \\
&= \begin{bmatrix} -1 & 4 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} \\
&= \begin{bmatrix} -1 & 4 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} -3 & 0 \\ 0 & -2 \end{bmatrix}
\end{aligned}$$

$$\Rightarrow \quad \lambda_1 = -3$$
$$\lambda_2 = -2$$

c) Does the estimation error vanish for $t \to \infty$? Justify your answer.

(ca. 1 point)

$$\mathrm{Re}(\lambda_{1,2}) < 0 \quad \Rightarrow \quad \lim_{t\to\infty} \tilde{\mathbf{x}} = \mathbf{0}$$

Task 8: **Transfer Function** *(ca. 3 points)*

Calculate the transfer function $G(s) = \frac{X(s)}{F(s)}$ for the system of second order differential equations

$$\mathbf{M}\,\ddot{\mathbf{q}} = -\mathbf{R}\,\dot{\mathbf{q}} + \mathbf{e}\,f$$

with the output equation $x = \mathbf{a}^\mathsf{T}\,\mathbf{q}$.
Hint: $\mathbf{M} \in \mathbb{R}^{n\times n},\ \mathbf{R} \in \mathbb{R}^{n\times n},\ \mathbf{q}(t) \in \mathbb{R}^{n\times 1},\ \mathbf{e} \in \mathbb{R}^{n\times 1},\ \mathbf{a}^\mathsf{T} \in \mathbb{R}^{1\times n},\ f(t) \in \mathbb{R},\ x(t) \in \mathbb{R}$

$$\mathbf{M}\,s^2\,\mathbf{Q}(s) + \mathbf{R}\,s\,\mathbf{Q}(s) = e\,F(s) \qquad \text{(1P)}$$

$$(\mathbf{M}\,s^2 + \mathbf{R}\,s)\,\mathbf{Q}(s) = e\,F(s)$$

$$\mathbf{Q}(s) = (\mathbf{M}\,s^2 + \mathbf{R}\,s)^{-1}\,e\,F(s) \qquad \text{(1P)}$$

$$X(s) = \mathbf{a}^\mathsf{T}\,\mathbf{Q}(s)$$

$$X(s) = \underbrace{\mathbf{a}^\mathsf{T}\,(\mathbf{M}\,s^2 + \mathbf{R}\,s)^{-1}\,e}_{G(s)}\,F(s) \qquad \text{(1P)}$$

Task 9: Control Design / State Feedback *(ca. 6 points)*

Consider the following system with state vector $\mathbf{x}$, control input u and output signal y:

$$\dot{\mathbf{x}} = \underbrace{\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}}_{\mathbf{A}} \mathbf{x} + \underbrace{\begin{bmatrix} 0 \\ -1 \end{bmatrix}}_{\mathbf{b}} u \qquad y = \underbrace{\begin{bmatrix} 5 & 0 \end{bmatrix}}_{\mathbf{c}^\mathsf{T}} \mathbf{x}$$

a) Calculate the feedback gain $\mathbf{r}$ of the following control law such that the closed-loop eigenvalues become $\lambda_{R,1}, \lambda_{R,2}$.

$$u = -\mathbf{r}^\mathsf{T}\mathbf{x}, \quad \mathbf{r} \in \mathbb{R}^2, \quad \lambda_{R,1} = -2, \quad \lambda_{R,2} = -3$$

(ca. 2 points)

$$\mathbf{r} = [r_1, r_2]^\mathsf{T}$$

$$\mathbf{A}_R = \mathbf{A} - \mathbf{b}\mathbf{r}^\mathsf{T}$$

$$(s\mathbf{I} - \mathbf{A}_R) = \begin{bmatrix} s+1 & -1 \\ 1 - r_1 & s - r_2 + 1 \end{bmatrix}$$

$$\det(s\mathbf{I} - \mathbf{A}_R) = s^2 + (2 - r_2)s + (2 - r_2 - r_1) \qquad \text{(1P)}$$

$$\stackrel{!}{=} \prod_k (s - \lambda_{R,k}) = s^2 + 5s + 6$$

$$(2 - r_2) = 5 \quad \Rightarrow \quad r_2 = -3$$

$$(2 - r_2 - r_1) = 6 \quad \Rightarrow \quad r_1 = -1$$

$$\Rightarrow \mathbf{r} = [-1, -3]^\mathsf{T} \qquad \text{(1P)}$$

b) The output signal y shall track the reference signal w with steady-state accuracy. Calculate the corresponding parameter m_u of the following control law.

$$u = -\mathbf{r}^\mathsf{T}\,\mathbf{x} + m_u\,w, \qquad m_u \in \mathbb{R}, \qquad \mathbf{r} = [-4, -2]^\mathsf{T}$$

(ca. 4 points)

$$\mathbf{A}_w = \mathbf{A} - \mathbf{b}\mathbf{r}^\mathsf{T}$$

$$\mathbf{b}_w = \mathbf{b}\,m_u$$

$$(s\mathbf{I} - \mathbf{A}_w) = \begin{bmatrix} s+1 & -1 \\ 5 & s+3 \end{bmatrix} \qquad \text{(1P)}$$

$$(s\mathbf{I} - \mathbf{A}_w)^{-1} = \frac{1}{s^2 + 4s + 8}\begin{bmatrix} s+3 & 1 \\ -5 & s+1 \end{bmatrix} \qquad \text{(1P)}$$

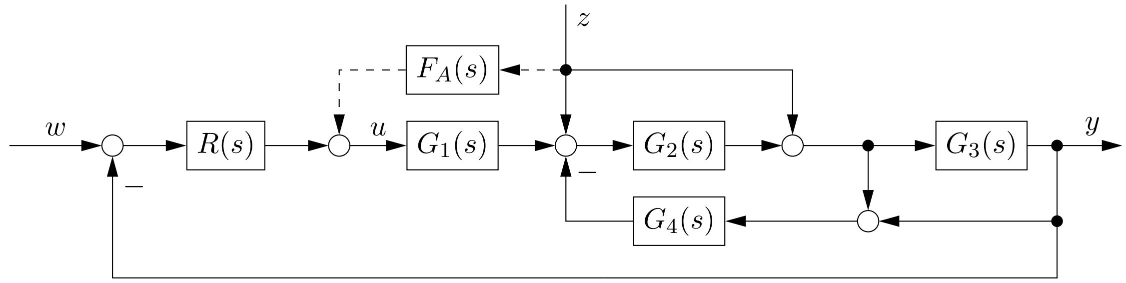
$$G_{yw}(s) = \mathbf{c}^\mathsf{T}(s\mathbf{I} - \mathbf{A}_w)^{-1}\mathbf{b}_w = \frac{-5\,m_u}{s^2 + 4s + 8} \qquad \text{(1P)}$$

$$1 \stackrel{!}{=} \lim_{s \to 0} G_{yw}(s) = -\frac{5}{8}m_u \qquad \text{(0.5P)}$$

$$\Rightarrow m_u = -\frac{8}{5} \qquad \text{(0.5P)}$$

Task 10: Control Structures / Disturbance Rejection *(ca. 5 points)*

Consider the following system with reference signal w, control input u, output signal y, plant $G_1(s) \ldots G_4(s)$ and controller $R(s)$. The system is affected by the disturbance z, which is measured. A disturbance rejection filter $F_A(s)$ shall be designed.



a) Calculate the *open-loop* transfer functions $G_{yu}(s)$ and $G_{yz}(s)$ from the control input and disturbance to the output. Here, $F_A(s) = 0$. *(ca. 2 points)*

Auxiliary cut v at the input to G_3:

$$v = (1 + G_2)z + G_1 G_2 u - G_2 G_4 (1 + G_3) v$$

$$\Rightarrow \quad G_{vu}(s)|_{z=0} = \frac{G_1 G_2}{1 + G_2 G_4 (1 + G_3)}$$

$$\Rightarrow \quad G_{vz}(s)|_{u=0} = \frac{1 + G_2}{1 + G_2 G_4 (1 + G_3)}$$

$$G_{yv} = G_3$$

$$\Rightarrow \quad G_{yu}(s) = \frac{G_1 G_2 G_3}{1 + G_2 G_4 (1 + G_3)} \qquad \text{(1P)}$$

$$\Rightarrow \quad G_{yz}(s) = \frac{(1 + G_2) G_3}{1 + G_2 G_4 (1 + G_3)} \qquad \text{(1P)}$$

b) Using the following transfer functions, calculate an appropriate disturbance rejection filter $F_A(s)$ that can be implemented in practice. If necessary, apply a filter of the form $V(s)$ (give reasons for this!).
Note: This task can be solved independently.

$$G_{yu} = \frac{s-2}{(s+2)(3s^2+13s-8)} \qquad G_{yz} = \frac{2s+1}{3s^2+13s-8}$$

$$V(s) = \frac{(s-a)^m}{(Ts+1)^n} \qquad T = 0.01,\ a \in \mathbb{R},\ m,n \in \mathbb{Z}_{\geq 0}$$

(ca. 3 points)

$$u = F_A z$$

$$y = G_{yu}u + G_{yz}z = (G_{yu}F_A + G_{yz})z \overset{!}{=} 0\ \forall z$$

$$\Rightarrow \quad F_{A,ideal} = -G_{yu}^{-1}G_{yz} = \frac{(s+2)(2s+1)}{s-2} \qquad \text{(1P)}$$

$F_{A,ideal}$ is *unstable* and *violates causality*. We need to cancel the unstable pole $s_1 = 2$ with a right half-plane zero and use a second-order low-pass filter to restore causality.(1P)

$$F_A = \frac{s-2}{(s+1)^2}F_{A,ideal} = \frac{(s+2)(2s+1)}{(Ts+1)^2} = 2\frac{(s+2)(s+0.5)}{(0.01s+1)^2} \qquad \text{(1P)}$$

Task 11: Discrete-Time Control *(ca. 3 points)*

For the implementation of a discrete time filter on a digital micro controller the continuous time transfer function $G(s)$ shall be discretized and the difference equation of the discrete time filter shall be derived.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{20}{(s+10)}$$

a) Derive the discrete time transfer function $R(z) = \frac{Y(z)}{U(z)}$. The sampling frequency of the discrete time filter is $f_T = 5$ Hz. Use the following transformation:

$$s = \frac{1}{T}(1 - z^{-1})$$

(ca. 1 point)

$$R(z) = \frac{20}{\frac{1}{T}(1-z^{-1}) + 10} = \frac{20T}{(1-z^{-1}) + 10T}$$

$$R(z) = \frac{4}{3 - z^{-1}} = \frac{4\,z}{3\,z - 1}$$

b) Derive the difference equation for the computation of the filter output signal in the following form:

$$y[k] = f(u[k], u[k-1], \ldots, y[k-1], \ldots) \; .$$

(ca. 1 point)

$$R(z) = \frac{4}{3 - z^{-1}}$$
$$3\,y[k] - z^{-1}\,y[k] = 4\,u[k]$$
$$y[k] = \frac{1}{3}(y[k-1] + 4\,u[k])$$

c) At the time step $k = 1$, a unit step signal is applied at the filter input: $u[k > 0] = 1$. The initial output value of the filter is $y[k = 0] = 4$. What is the value of the filter output signal $y[k]$ at the time step $k = 2$? *(ca. 1 point)*

$$y[0] = 4$$
$$y[1] = \frac{1}{3}(4 + 4 \cdot 1) = \frac{8}{3}$$
$$y[2] = \frac{1}{3}(\frac{8}{3} + 4 \cdot 1) = \frac{20}{9}$$

Task 12: Short Questions *(ca. 3 points)*

Tick the correct answer.

a) Which of the following elements leads to steady-state accuracy?

Options	Antwort / Answer
Integrator	✓
Differentiator	
Low pass filter	
High pass filter	

(ca. 1 point)

b) The error signal of a standard feedback control system depends on:

Options	Answer
Command and output signal.	✓
Command and plant input signal	
Plant input and output variable.	
Model parameters.	

(ca. 1 point)

c) An open-loop control system has the following property:

Options	Answer
The input variables are dependent on the output signals.	
The input variables are independent of the output signals.	✓
The input variables depend on the size of the system.	
The input variables depend on plant parameters.	

(ca. 1 point)

Additional Pages: Make sure to uniquely assign the notes to a specific subtask!

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