Test-Exam „Engineering Mechanics I+II“

You have 60 minutes.

Exam: 13 pages.

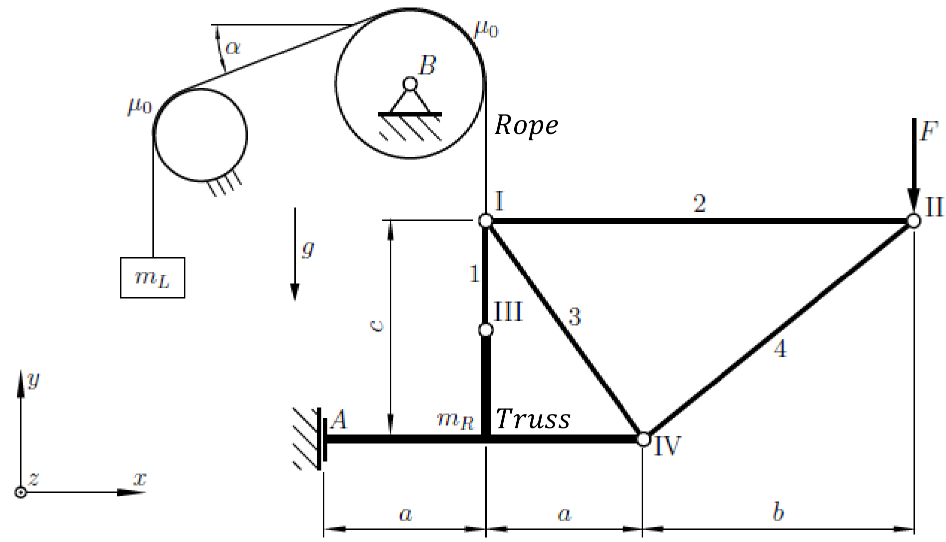
Please check the exam booklet for completeness. The tasks can be performed in any order.

Last name	
First name	
Application number	

	Reached credits / Maximum reachable credits
Points	/ 60

Calculation Task 1

10



The illustrated planar system is in static equilibrium. A frame (mass m_R) is vertically movable in A and is mounted rigidly against bending. The frame is mounted in III and IV with a massless ideal truss, consisting of members 1 to 4, connected with hinges. Node II of the truss is loaded vertically by the external force F. A massless flexible rope is attached to node I. The cable is routed around two cylindrical rollers. The right-hand roller is supported at its centre B by a fixed bearing. The left roller is firmly connected to the surroundings. Between the rope and the rollers there is static friction (static friction coefficient μ_0). There is a load (mass m_L) attached to the end of the rope. The rope force acting on node I is $W > 0$. The acceleration due to gravity g acts.

Given: $\alpha, \mu_0, a, b, c, F, g, m_R$

a) Calculate the rope force W that acts on node I.

Force balance in vertical direction for the system
frame + truss

$\sum F_y = 0$: $\quad m_R g + F - W = 0$

$W = m_R g + F$

$\sum$	

b) Calculate the member forces $S1, S2, S3$ and $S4$ of members 1 to 4 using the convention that member forces of members loaded in tension are positive. For this subtask, take the rope force W as a given.

Node II:
Angle which is enclosed by member 4 with the horizontal: $\alpha = atan\frac{c}{b}$

$$\sum F_x = 0: \; -S_2 - S_4\cos(\alpha) = 0$$

$$\sum y = 0: \; -F - S_4\sin(\alpha) = 0$$

So that:

$$S_4 = -\frac{F}{\sin(\alpha)}$$

$$S_2 = -\frac{F}{\tan(\alpha)}$$

Node I:
Angle which is enclosed by member 3 with the horizontal: $\beta = atan\frac{c}{a}$

$$\sum F_x = 0: S_2 + S_3\cos(\beta) = 0$$

$$\sum y = 0: W - S_1 - S_3\sin(\alpha) = 0$$

So that:

$$S_3 = -\frac{S_2}{\cos(\alpha)}$$

$$S_1 = W - S_3\sin(\beta)$$

$$S_1 = W + F\frac{\tan(\beta)}{\tan(\alpha)}$$

$$S_2 = \frac{F}{\tan(\alpha)}$$

$$S_3 = -\frac{F}{\tan(\alpha)\cos(\beta)}$$

$$S_4 = -\frac{F}{\sin(\alpha)}$$

c) Calculate the minimum mass $m_{l,min}$ of the load for which the system is in static equilibrium. For this subtask, take the rope force W as given.

$\sum$	

Euler-Eytelwein

$$m_{L,min}\, g = e^{-\mu_0 \varphi W}$$

Angle of rope contact:

$$\varphi = \frac{\pi}{2} - \alpha$$

$$m_{L,min} = \frac{W}{g}\, e^{-\left(\mu_0\left(\frac{\pi}{2}-\alpha\right)\right)}$$

$\sum$

Calculation Task 2

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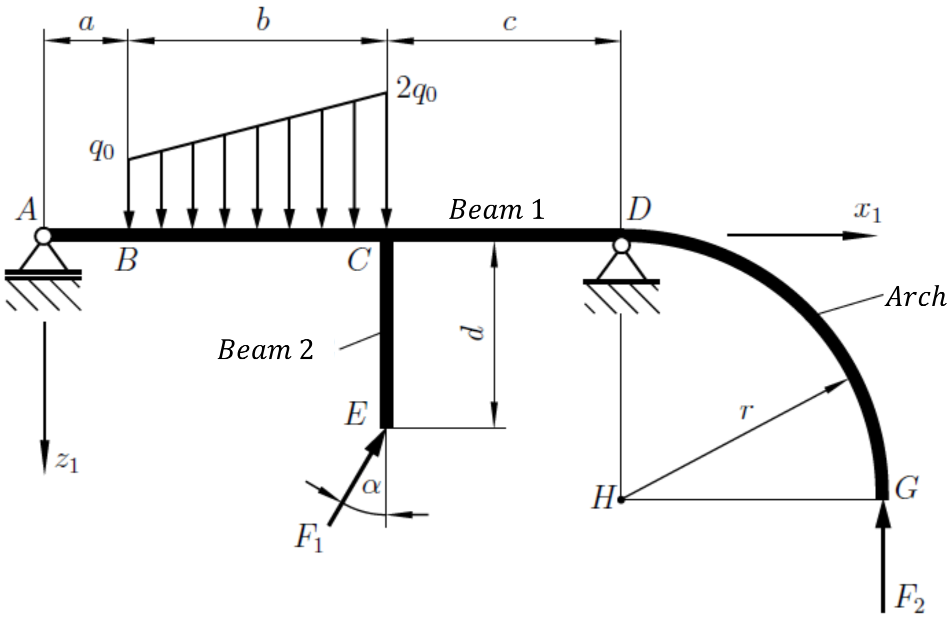


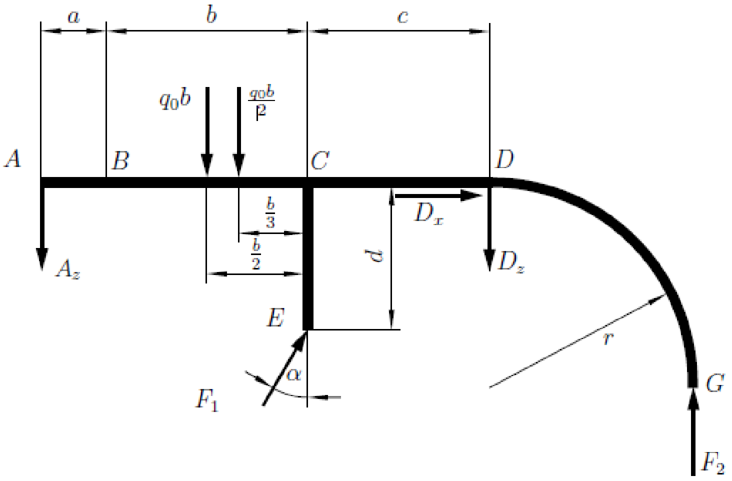
The planar massless frame shown consists of beams 1 (length $a + b + c$) and 2 (length d), which are connected to each other in C, and an arch, which is connected to beam 1 in D. The arch is described by a quarter circle (radius r, center H). The frame is supported by a horizontally movable floating bearing in A and a fixed bearing in D. The frame is loaded by force F_1 at E and force F_2 at G. Beam 1 is loaded between B and C with a line load that increases from q_0 to $2q_0$. The bearing forces acting on the frame in A and D are calculated in the given coordinate system through the vectors $F_A = (0\ \ 0\ \ A_z)^T$ and $F_D = (D_x\ \ 0\ \ D_z)^T$.

Given: $\alpha, a, b, c, d, F_1, F_2, q_0, r$

a) Set up three linearly independent equations from which the bearing reactions A_z, D_x and D_z can be determined.

$\sum$

Force and moment section:



Balance of forces and moments:

$$\sum M^D = 0: \quad A_z(a+b+c) + \frac{q_0 b}{2}\left(c + \frac{1}{3}b\right) + + q_0 b\left(c + \frac{1}{2}b\right) - \cos\alpha F_1 c + \sin\alpha F_1 d + F_2 r = 0$$

$$\sum M^A = 0: \quad -q_0 b(a + \frac{1}{2}b) - \frac{q_0 b}{2}(a + \frac{2}{3}b) + F_1 \cos\alpha(a+b) + +F_1 \sin\alpha d - D_z(a+b+c) + F_2(a+b+c+r) = 0$$

$$\sum F_{horizontal} = 0: \quad D_x + \sin\alpha F_1 = 0$$

$$\sum F_{vertikal} = 0: \quad A_z + q_0 b + \frac{q_0 b}{2} - \cos\alpha F_1 + D_z - F_2 = 0$$

$$A_z = -\frac{1}{a+b+c}\left(\frac{q_0 b}{2}\left(c + \frac{1}{3}b\right) + q_0 b\left(c + \frac{1}{2}b\right) - \cos\alpha F_1 c + \sin\alpha F_1 d + F_2 r\right)$$

$$D_x = -\sin\alpha F_1$$

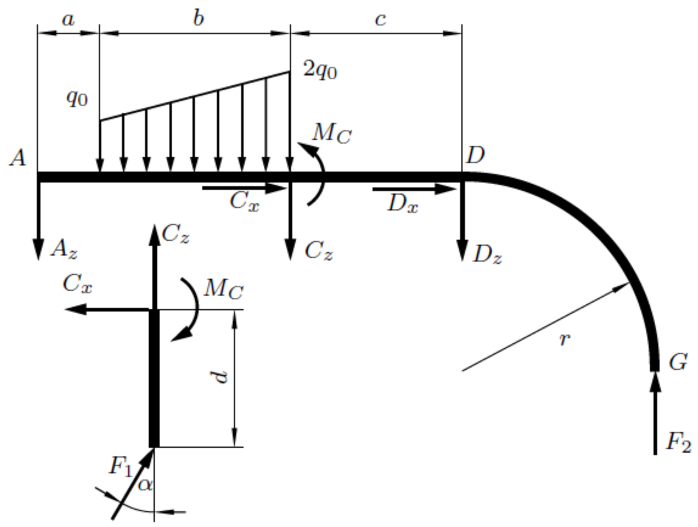
$$D_z = -A_z - q_0 b - \frac{q_0 b}{2} + \cos\alpha F_1 + F_2$$

$\sum$

b) Specify the distribution of normal force $N_1(x_1)$, line load $q_1(x_1)$, shear force $Q_1(x_1)$ and bending moment $M_1(x_1)$ in beam 1 for the range $0 \leq x1 < a+b+c$ using the Foppl notation. For this subtask, take the bearing reaction A_z as given.

Do not use your results from subtask a) here
Additionally given: A_z

Force and moment section:



Balance of forces and moments at beam 2:

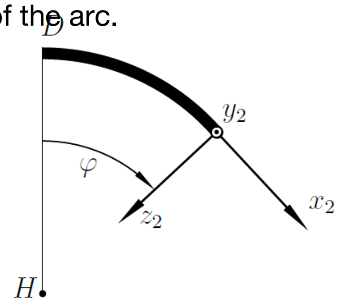
$$\sum F_{vertikal} = 0: \quad -C_z - \cos\alpha F_1 = 0 \qquad \Rightarrow C_z = -\cos\alpha F_1$$
$$\sum F_{horizontal} = 0: \quad -C_x + \sin\alpha F_1 = 0 \qquad \Rightarrow C_x = \sin\alpha F_1$$
$$\sum M^C = 0: \quad -M_C + \sin\alpha F_1 d = 0 \qquad \Rightarrow M_C = \sin\alpha F_1 d$$

$$N_1(x_1) = -|C_x\langle x_1 - (a+b)\rangle^0$$
$$q_1(x_1) = q_0\langle x_1 - a\rangle^0 + \frac{q_0}{b}\langle x_1 - a\rangle^1 - 2q_0\langle x_1 - (a+b)\rangle^0 - \frac{q_0}{b}\langle x_1 - (a+b)\rangle^1$$
$$Q_1(x_1) = -q_0\langle x_1 - a\rangle^1 - \frac{q_0}{2b}\langle x_1 - a\rangle^2 + 2q_0\langle x_1 - (a+b)\rangle^1 + \frac{q_0}{2b}\langle x_1 - (a+b)\rangle^2 - A_z + \cos\alpha F_1\langle x_1 - (a+b)\rangle^0$$
$$M_1(x_1) = -\frac{q_0}{2}\langle x_1 - a\rangle^2 - \frac{q_0}{6b}\langle x_1 - a\rangle^3 + q_0\langle x_1 - (a+b)\rangle^2 + \frac{q_0}{6b}\langle x_1 - (a+b)\rangle^3 - A_z x_1 + \cos\alpha F_1\langle x_1 - (a+b)\rangle^1 - \sin\alpha F_1 d\langle x_1 - (a+b)\rangle^0$$

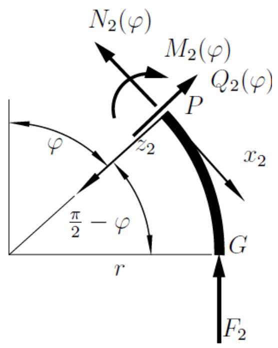
∑

c) Calculate the distribution of the normal force $N_2(\varphi)$, the lateral force $Q_2(\varphi)$ and the bending moment $M_2(\varphi)$ in the x_2, y_2, z_2 coordinate system shown along the arc between D and G as a function of the angle φ. The z_2-axis is always perpendicular to the center H of the arc.

Additionally given: φ



Force section:



Balance of force and moments:

$$\sum F_{x2} = 0: \quad -N_2(\varphi) - \cos\left(\frac{\pi}{2} - \varphi\right) F_2 = 0$$
$$\sum F_{z2} = 0: \quad -Q_2(\varphi) - \sin\left(\frac{\pi}{2} - \varphi\right) F_2 = 0$$
$$\sum M^P = 0: \quad -M_2(\varphi) + r\left(1 - \cos\left(\frac{\pi}{2} - \varphi\right)\right) F_2 = 0$$

Alternativ:

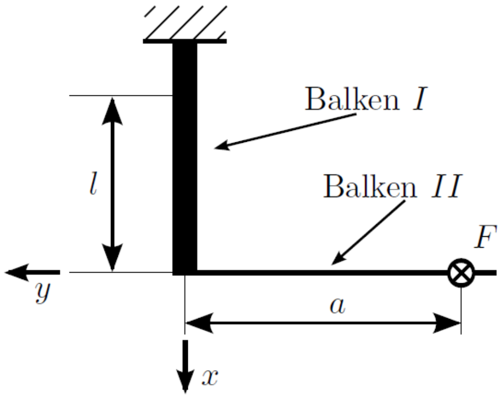
$$\sum F_{x2} = 0: \quad -N_2(\varphi) - \sin(\varphi) F_2 = 0$$
$$\sum F_{z2} = 0: \quad -Q_2(\varphi) - \cos(\varphi) F_2 = 0$$
$$\sum M^P = 0: \quad -M_2(\varphi) + r(1 - \sin(\varphi)) F_2 = 0$$

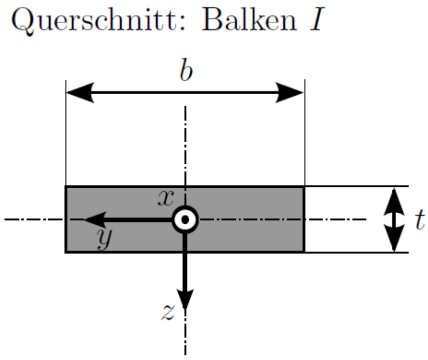
$$N_2(\varphi) = -\sin(\varphi) F_2$$
$$Q_2(\varphi) = -\cos(\varphi) F_2$$
$$M_2(\varphi) = r(1 - \sin(\varphi)) F_2$$

∑	

Calculation Task 3

10





A structure consists of a beam I (Young's modulus E) and a rigid beam II. The ends of beam I and beam II are connected flexurally rigid at the origin of the given coordinate system. The other end of beam I is firmly clamped. The cross section of beam I has the width b and the thickness t, where: $t \ll b$. The bar II is in z-direction loaded by the force F. At distance a from the origin is the point of application of Force F.

Given: a, b, l, t, E, F

a) Determine the moments of resistance of beam I against bending $W_{B,y}$ and torsion $W_{T,x}$.

$$W_{B,y} = \frac{I_y}{z_0}; \; I_y = \frac{b\,t^3}{12};$$

$$W_{T,x} = \frac{I_{T,x}}{t}; \; I_{T,x} = \frac{b\,t^3}{3}$$

$$W_{B,y} = \frac{1}{6} b t^2$$

$$W_{T,x} = \frac{1}{3} b t^2$$

b) Determine the shear stress τ_{xy} and the normal stress σ_x at position P(x=-l, y=0, z=-1/2).

The Shear stress origins due to the torsional moment M_T , the normal stress due to the bending moment M_B at the location l

$\sum$

$M_T = -a\,F$

$M_B = -l\,F$

Damit folgt:

$$\tau_{xy} = \frac{M_T t}{I_{T,x}} = -\frac{3\,a\,F}{bt^2}$$

$$z = -\frac{t}{2}$$

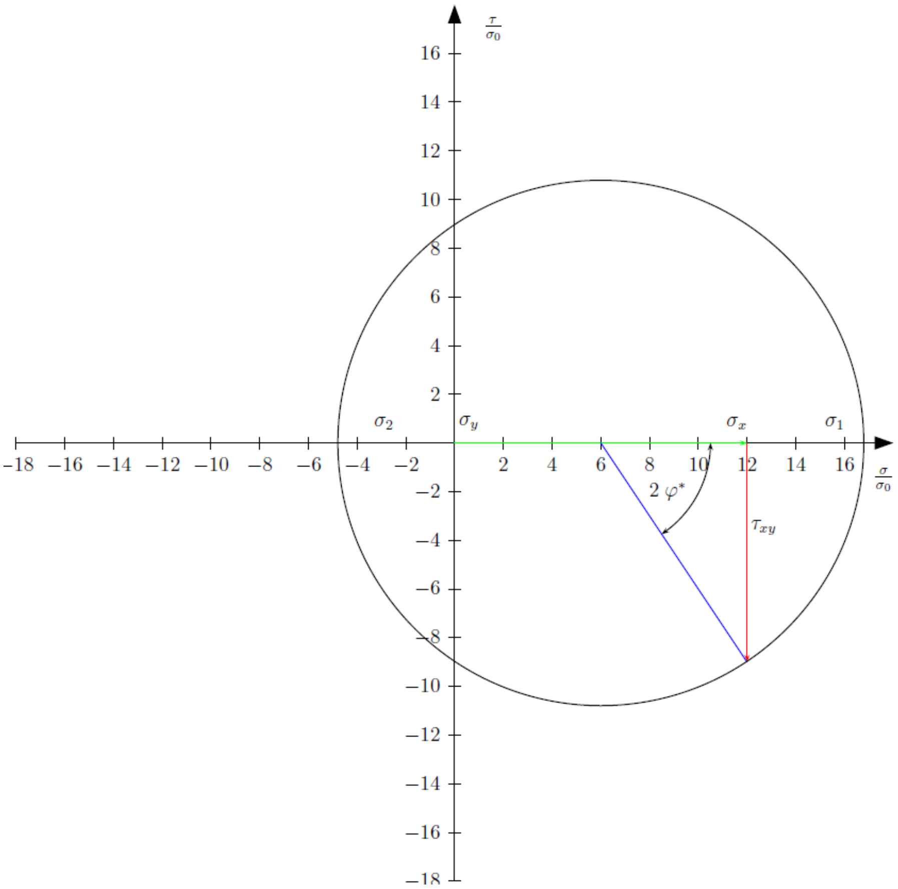
$$\sigma_x = \frac{M_B z}{I_y} = \frac{6\,F\,l}{b\,t^2}$$

$$\tau_{xy} = -\frac{3a}{bt^2}\,F$$

$$\sigma_x = \frac{6l}{bt^2}\,F$$

For the following subtask, the shear stress at the point P is to be calculated with $\tau_{xy} = -9\sigma_0$ and the normal stress with $\sigma_x = 12\sigma_0$.

c) Sketch Mohr's stress circle at location P. Mark the principal stresses σ_1 and σ_2 and the angle φ^*, measured between the x-axis and the direction of the main stress σ_1.



$\sum$

Calculation Task 4

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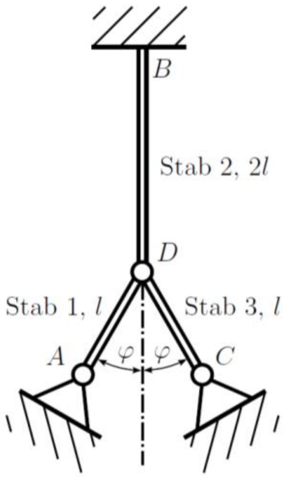
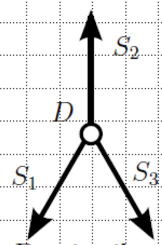
The flat rod system shown on the right is made up of the homogeneous slender rods 1 to 3. The rods 1 and 3 have a length of l, and rod 2 a length of $2l$. All rods have the cross-sectional area A, the moment of inertia I, the modulus of elasticity E and the coefficient of thermal expansion α_T . The rods 1 and 3 are supported in A and C respectively via fixed bearings. Rod 2 is firmly clamped in B. At D, rods 1 to 3 are articulately jointed to each other. The figure shows the stress-free initial state of the system at room temperature T_R. Outgoing from this state, the temperature of all rods of the system is slowly and evenly increased until the system reaches the critical temperature at which it fails due to buckling.



Given: $l, A, E, I, \alpha_T, T_R$

a) Determine the angle $\varphi = \varphi_1$, for which all rods reach the critical temperature simultaneously and buckle.

Kräftegleichgewichte in D:



$$\sum F_H = 0 : S_1 = S_3$$

$$\sum F_V = 0 : S_2 = 2S_1 \cos\varphi \qquad (4.1)$$

Bei Stab 2 liegt Knickfall III vor, bei den Stäben 1 und 3 Knickfall II. Damit gilt

$$S_{2,\text{krit}} = -(1,43)^2 \frac{\pi^2 EI}{4l^2}$$

$$S_{1,\text{krit}} = S_{3,\text{krit}} = -\frac{\pi^2 EI}{l^2}.$$

Im Grenzfall versagen die Stäbe gleichzeitig:

$$S_1 = S_{1,\text{krit}}$$

$$S_2 = S_{2,\text{krit}}$$

In (4.1):

$$(1,43)^2 \frac{\pi^2 EI}{4l^2} = 2\frac{\pi^2 EI}{l^2} \cos\varphi_1$$

$$\cos\varphi_1 = \frac{(1,43)^2}{8}$$

$$\varphi_1 = \operatorname{acos}\left(\frac{1{,}43^2}{8}\right)$$

∑	

b) In this part the angle $\varphi = \varphi_2$ is chosen in a way that rod 2 buckles before the rods 1 and 3 due to the increase in temperature. Determine the critical temperature T_{crit} at which rod 2 buckles. Consider the angle φ_2 as given.

General formulation for the change of length of the rods

$$\Delta l = \frac{Sl}{EA} + l\alpha_T \Delta T$$

Change in length of the rods

$$\Delta l_1 = \frac{S_1 l}{EA} + l\alpha_T \Delta T \quad (4.2)$$

$$\Delta l_2 = 2\frac{S_2 l}{EA} + 2l\alpha_T \Delta T \quad (4.3)$$

Geometric tolerability

$$\cos\varphi_2 = -\frac{\Delta l_1}{\Delta l_2} \quad (4.4)$$

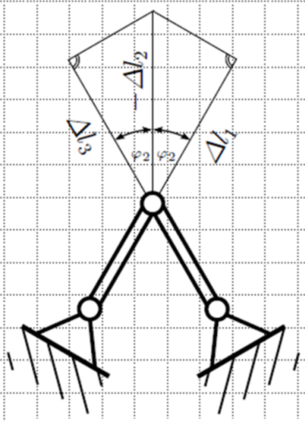
Insert equation (4.2) and (4.3) in (4.4):

$$\cos\varphi_2 = -\frac{S_1 + EA\alpha_T \Delta T}{2S_2 + 2EA\alpha_T \Delta T}$$

$$S_1 = -2\cos\varphi_2 S_2 - EA\alpha_T \Delta T(1 + 2\cos\varphi_2)$$

Insert in (4.1)

$$S_2 = -\frac{2EA\alpha_T \Delta T \cos\varphi_2(1 + 2\cos\varphi_2)}{1 + 4\cos^2\varphi_2}$$



Standard buckling case 3, therefor:

$$S_{2,\text{krit}} = -(1,43)^2 \frac{\pi^2 EI}{4l^2}$$

$$\frac{2EA\alpha_T \Delta T_{\text{krit}} \cos\varphi_2(1 + 2\cos\varphi_2)}{1 + 4\cos^2\varphi_2} = (1,43)^2 \frac{\pi^2 EI}{4l^2}$$

$$\Delta T_{\text{krit}} = \frac{(1,43)^2 \pi^2 I\,(1 + 4\cos^2\varphi_2)}{8A\alpha_T \cos\varphi_2(1 + 2\cos\varphi_2) l^2}$$

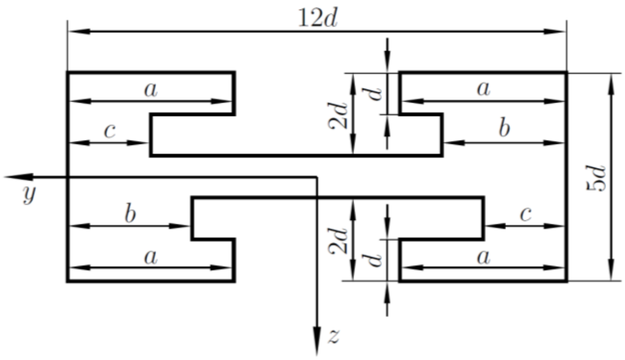
$\sum$

$$T_{crit} = T_R + 1.43^2\pi^2 I \frac{1+4\cos^2\varphi_2}{8A\alpha_T \cos\varphi_2 (1+2\cos\varphi_2) l^2}$$

Calculation Task 5

6

Given is the illustrated point-symmetrical cross-sectional area of a beam. The given coordinate system is located in the center of gravity of the cross-section. In the initial configuration shown, the relations between the dimensions are given as $a = 4d$, $b = 3d$ and $c = 2d$. The moments of inertia I_y and I_z as well as the surface moment of deviation I_{yz} of the cross-sectional surface are examined.



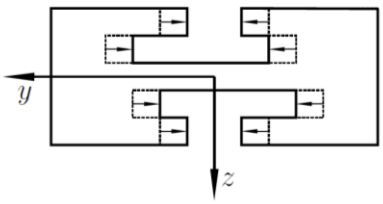
Given: d

a) Indicate whether the specified values in the initial configuration are positive (+) or equal to zero (0) or negative (-)

	+	0	−
I_y	☒	☐	☐
I_z	☒	☐	☐
I_{yz}	☒	☐	☐

b) In this part the lengths a, b and c are increased in comparison to the initial configuration by an additional length d. Indicate whether the specified values increase (+), decrease (-) or remain unchanged (=) in comparison to the initial configuration.

	+	=	−
I_y	☒	☐	☐
I_z	☒	☐	☐
I_{yz}	☐	☐	☒



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