Mock Exam for Engineering Mechanics 1 and 2

Exam: xxxxxxx / Mock Exam    Date: xxxxxxxxxxxxxxx xxxxxxx0th, 0
Examiner: xxxx                  Time: xxxx00:00 – xxxx00:00

Working instructions

• This exam consists of 14 pages with a total of 6 problems. Please make sure now that you received a complete copy of the exam.

• The total amount of achievable credits in this exam is 60 credits.

• Detaching pages from the exam is prohibited.

• Allowed resources:
  – one non-programmable pocket calculator
  – one analog dictionary English ↔ native language

• Subproblems marked by * can be solved without results of previous subproblems.

• Answers are only accepted if the solution approach is documented. Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.

• Do not write with red or green colors nor use pencils.

• Physically turn off all electronic devices, put them into your bag and close the bag.
For multiple choice problems mark the correct answers as follows:

Mark correct answers with a cross
To undo a cross, completely fill out the answer option
To re-mark an option, use a human-readable marking

Correct answers result in positive credit, wrong answers will result in negative credit, empty boxes are not counted. It is not possible to get a total score that is negative.
Problem 1  (6 credits)

a)*

For the shown cross sections and the given coordinate, which statement is true for the second moment of area $I_{yy}$?

- $I = III > II$
- $II > I > III$
- $I = III < II$
- $I > III > II$

b)*

Which statement regarding the second moments of area is true?

- $I_{yz}$ can be positive, negative or zero
- For every cross section of a beam: $I_{zz} > I_{yy}$
- $I_{yz}$ can be positive and negative, but never zero
- For every cross section of a beam: $I_{yy} > I_{zz}$

c)*

The shown bar is heated uniformly by $\Delta T$. The thermal expansion coefficient is constant ($\alpha_T$). No other loads are applied. The displacement function $u(x)$ will be:

- Constant
- Quadratic
- Cubic
- $x$ Linear
For the shown cross sections, which statement is true? ($I_{yy}$ and $I_{zz}$ are second moment of areas about the respective axis and $I_T$ is the torsion constant of the cross-section)

- $I_{yy,1} > I_{yy,2}$; $I_{zz,1} < I_{zz,2}$; $I_T,1 < I_T,2$
- $I_{yy,1} = I_{yy,2}$; $I_{zz,1} < I_{zz,2}$; $I_T,1 < I_T,2$
- $I_{yy,1} > I_{yy,2}$; $I_{zz,1} < I_{zz,2}$; $I_T,1 > I_T,2$

- $I_{yy,1} < I_{yy,2}$; $I_{zz,1} > I_{zz,2}$; $I_T,1 < I_T,2$
- $I_{yy,1} > I_{yy,2}$; $I_{zz,1} < I_{zz,2}$; $I_T,1 = I_T,2$
- $I_{yy,1} > I_{yy,2}$; $I_{zz,1} = I_{zz,2}$; $I_T,1 = I_T,2$
Shown is the thin-walled cross section of a bar. The bar is loaded by a torsional moment. At which point would one expect the greatest magnitude of the torsional shear stress $|\tau|$?

- D
- C
- B
- A

$t_3 > t_2 > t_1$
Problem 2  Plane truss (19 credits)

Given is the shown plane truss made of rigid bars. It is supported in the points A and B. The system is loaded by point loads at points C, D, E, G, and H. Bearing conditions, loads and dimensions are shown in the drawing.

Given quantities: a, F, coordinate system xy (right-hand system)

All subtasks can be solved independently of each other.

a)* Is the truss statically determined?

☐ No  ❌ Yes

b)* How many degrees of freedom are constrained by the support in A?

☒ 2  ☐ 0  ☐ 1  ☒ 3

c)* Calculate the support forces $A_x$. Consider the constant $k = 2$ in point G.

☒ $A_x = -4.5F$  ☐ $A_x = 7.0F$

☒ $A_x = -9.0F$  ☒ $A_x = 3.5F$

d)* Calculate the support forces $A_y$. Consider the constant $k = 2$ in point G.

☐ $A_y = -2.5F$  ☒ $A_y = 8.0F$

☒ $A_y = -6.0F$  ☐ $A_y = 4.0F$
e)* Calculate the support forces $B_x$. Consider the constant $k = 2$ in point G.

- $B_x = 0\text{F}$
- $B_x = 4.5\text{F}$
- $B_x = 3.0\text{F}$
- $B_x = 6.5\text{F}$

f)* Calculate the bar force $S_1$ in bar 1. Consider the constant $k = 4$ in point G.

Given: The support forces are given with the values being $A_x = -\frac{15}{2}\text{F}$, $A_y = 6\text{F}$ and $B_x = \frac{15}{2}\text{F}$.

Convention: The positive direction of the support forces is aligned with the positive direction of the coordinate system axes.

- $S_1 = -7.0\text{F}$
- $S_1 = -6.0\text{F}$
- $S_1 = -5.0\text{F}$
- $S_1 = -4.0\text{F}$

g)* Calculate the bar force $S_2$ in bar 2. Consider the constant $k = 4$ in point G.

Given: The support forces are given with the values being $A_x = -\frac{15}{2}\text{F}$, $A_y = 6\text{F}$ and $B_x = \frac{15}{2}\text{F}$.

Convention: The positive direction of the support forces is aligned with the positive direction of the coordinate system axes.

- $S_2 \approx 3.58\text{F}$
- $S_2 \approx -12.73\text{F}$
- $S_2 \approx -9.41\text{F}$
- $S_2 \approx 7.21\text{F}$

h)* Calculate the bar force $S_3$ in bar 3. Consider the constant $k = 4$ in point G.

Given: The support forces are given with the values being $A_x = -\frac{15}{2}\text{F}$, $A_y = 6\text{F}$ and $B_x = \frac{15}{2}\text{F}$.

Convention: The positive direction of the support forces is aligned with the positive direction of the coordinate system axes.

- $S_3 = 3.0\text{F}$
- $S_3 = -5.0\text{F}$
- $S_3 = -9.0\text{F}$
- $S_3 = 4.0\text{F}$
Problem 3  (6 credits)

Given is the shown planar system. The massless, rigid bar (length $3l$) is supported at points A and B and is support against the rigid environment at point B by a linear, mass-free spring (spring stiffness $k$). Two point masses $m$ and $m$ and are attached to the bar. The position of the bar is completely and unambiguously described by the angular coordinate $\phi$. The linear spring is relaxed in the position $\phi = \frac{\pi}{2}$. The acceleration due to gravity $g$ acts in the negative y-direction. Bearing conditions, loads as well as dimensions can be taken from the drawing.

Given quantities: $l$, $k$, $m$, $g$, coordinate systems $xy$ (right-handed systems), Angular coordinate $\phi \in [0, \frac{\pi}{2}]$

The system has two equilibrium positions. Cross the two correct equilibrium conditions.

- $\phi = \frac{2\pi}{3}$
- $\phi = \frac{\pi}{3}$
- $\phi = \frac{\pi}{4}$
- $\phi = \arcsin \frac{mg}{3kl}$
- $\phi = \arcsin \frac{3mg}{2kl}$
Problem 4  Cross-Section Properties (8 credits)

Shown is the cross section of a beam with measurement $a$.

**Given:** $a$, $(\bar{y}, \bar{z})$-coordinate-system

a)* Determine the coordinate $\bar{y}_c$ of the cross-section's centroid with respect to the given $(\bar{y}_c, \bar{z}_c)$-coordinate-system.

- $\bar{y}_c = 2.41a$  
- $\bar{y}_c = 2.19a$

b)* Determine the coordinate $\bar{z}_c$ of the cross-section's centroid with respect to the given $(\bar{y}_c, \bar{z}_c)$-coordinate-system.

- $\bar{z}_c = 2.52a$  
- $\bar{z}_c = 2.19a$

- $\bar{z}_c = 2.14a$  
- $\bar{z}_c = 2.41a$

C)* For this subtask, use the rounded centroid coordinates $\bar{y}_c = 2.2a$ and $\bar{z}_c = 2.15a$. Compute the second moment of area $I_{yy}$ with respect to the centroidal $(y,z)$-coordinate-system.

- $I_{yy} = 19.32a^4$  
- $I_{yy} = 18.71a^4$

- $I_{yy} = 20.43a^4$  
- $I_{yy} = 17.71a^4$
Problem 5  Beam Stresses (15 credits)

The shown clamped beam of length \( l \) is subjected to the moment \( M \) about the \( y \)-axis and the normal force \( F_1 \) at its free end in loadcase 1 and to the force \( F_2 \) in loadcase 2. The cross-section and the modulus of elasticity are constant along the beam's \( x \)-axis. Each subtask is referring to only one of both loadcases. Use the given coordinate system with its origin in the cross sections centroid \( C \).

Given: \( I_{zz} = 8748 \text{ mm}^4 \), \((x, y, z)\)-coordinate-system, \( a = 18 \text{ mm}, l = 1 \text{ m}, M = 40 \text{ kNmm}, F_1 = 10 \text{ kN}, F_2 = 20 \text{ kN} \)

\((y, z)\)-coordinates of points A, B, D and E: A: \((a^2, -a^2)\); B: \((a^2, a^2)\); D: \((-a^4, a^2)\); E: \((-a^4, 0)\)

a) Loadcase 1: Calculate the normal stress \( \sigma_{xx} \) at point A at \( x = \frac{l}{2} \)

- \( \sigma_{xx,A} = 72.03 \text{ MPa} \)
- \( \sigma_{xx,A} = 10.29 \text{ MPa} \)
- \( \sigma_{xx,A} = -72.03 \text{ MPa} \)
- \( \sigma_{xx,A} = -10.29 \text{ MPa} \)

b) Loadcase 1: Calculate the normal stress \( \sigma_{xx} \) at point B at \( x = \frac{l}{2} \)

- \( \sigma_{xx,B} = -10.29 \text{ MPa} \)
- \( \sigma_{xx,B} = 10.29 \text{ MPa} \)
- \( \sigma_{xx,B} = 72.03 \text{ MPa} \)
- \( \sigma_{xx,B} = -72.03 \text{ MPa} \)

c) Loadcase 1: Which statement is correct?

- Point A is stress free
- Point A is in compression
- Point A is in tension

d) Loadcase 1: Which statement is correct?

- Point B is in tension
- Point B is stress free
- Point B is in compression

e) Loadcase 1: Calculate \( F_1 \), such that the whole beam is in tension at \( x = \frac{l}{2} \).

- \( F_1 > 13.33 \text{ N} \)
- \( F_1 > 13.33 \text{ kN} \)
- \( F_1 < 13.33 \text{ N} \)
- \( F_1 < 13.33 \text{ kN} \)

f)*

g) Loadcase 2: Calculate the shear stress \( \tau_{xz} \) at point E at \( x = \frac{l}{2} \)
\[ \tau_{xz,E} = 0 \]

\[ \tau_{xz,E} = 92.59 \text{ MPa} \]

\[ \tau_{xz,E} = 185.18 \text{ MPa} \]

\[ \tau_{xz,E} = 30.86 \text{ MPa} \]
Problem 6  Energy Methods (6 credits)

Given:  \( l = 3\, \text{m} \);  \( q_0 = 150\, \text{N/m} \);  \( E = 210\, \text{GPa} = 210 \times 10^9\, \text{N/m}^2 \);  \( I_{yy} = 6 \times 10^{-8}\, \text{m}^4 \);  \( GA_s = \infty \)

a)* For the depicted system, the polynomial order of the bending moment  \( M(x) \) will be:

- [ ] Constant
- [ ] Quadratic
- [x] Cubic
- [ ] Linear

b)* Compute  \( M(x = l) \).

- [ ] \(-112.5\, \text{Nm}\)
- [x] \(-225\, \text{Nm}\)
- [ ] \(112.5\, \text{Nm}\)
- [ ] \(225\, \text{Nm}\)

c)* To calculate the deflection  \( w \) of the beam at  \( x = 0 \), one could use energy methods and apply a virtual force in the direction of the sought deflection. For the virtual "1"-System with the applied virtual force, the polynomial order of the bending moment  \( \bar{M}(x) \) would be:

- [ ] Quadratic
- [ ] Cubic
- [x] Linear
- [ ] Constant

d)* Compute  \( \bar{M}(x = l) \).

- [x] \(-3\, \text{Nm}\)
- [ ] \(3\, \text{Nm}\)
- [ ] \(1.5\, \text{Nm}\)
- [ ] \(-1.5\, \text{Nm}\)

e)* Compute the deflection  \( w \) at  \( x = 0 \).

- [ ] \(w = 0.0214\, \text{m}\)
- [ ] \(w = 0.0401\, \text{m}\)
- [x] \(w = 0.0442\, \text{m}\)
- [x] \(w = 0.0321\, \text{m}\)