## Note:

- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.


# Mock Exam for Engineering Mechanics 1 and 2 

Exam: $\quad x x x x x x x /$ Mock Exam Date: $x x x x x x x x x x x x y x x x y x x x x x 0^{\text {th }}, 0$<br>Examiner: xxxx<br>Time: xxxx00:00 - xxxx00:00



## Working instructions

- This exam consists of $\mathbf{1 4}$ pages with a total of $\mathbf{6}$ problems.

Please make sure now that you received a complete copy of the exam.

- The total amount of achievable credits in this exam is 60 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
- one non-programmable pocket calculator
- one analog dictionary English $\leftrightarrow$ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- Answers are only accepted if the solution approach is documented. Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.
$\qquad$
$\qquad$

For multiple choice problems mark the correct answers as follows:


Correct answers result in positive credit, wrong answers will result in negative credit, empty boxes are not counted. It is not possible to get a total score that is negative.

## Problem 1 (6 credits)

a) ${ }^{*}$


For the shown cross sections and the given coordinate, which statement is true for the second moment of area $I_{y y}$ ?
$\square I=I I I>I I$
$\square I I>I>I I$
$\square I=I I I<I I$
$\square I>I I I>I I$
b)*

Which statement regarding the second moments of area is true?
$\square I_{y z}$ can be positive, negative or zero
$\square$ For every cross section of a beam: $I_{z z}>I_{y y}$
$\square l_{y z}$ can be positive and negative, but never zero
$\square$ For every cross section of a beam: $I_{y y}>I_{z z}$
c) *


The shown bar is heated uniformly by $\Delta T$. The thermal expansion coefficient is constant $\left(\alpha_{T}\right)$. No other loads are applied. The displacement function $u(x)$ will be:
$\square$ Constant
$\square$ Cubic
$\square$ Quadratic
d) ${ }^{*}$

Cross-Section 1


Cross-Section 2


For the shown cross sections, which statement is true? ( $I_{y y}$ and $I_{z z}$ are second moment of areas about the respective axis and $I_{T}$ is the torsion constant of the cross-section)
$\square I_{y y, 1}>I_{y y, 2} ; I_{z z, 1}<I_{z z, 2} ; I_{T, 1}<I_{T, 2}$
$\square I_{y y, 1}<I_{y y, 2} ; I_{z z, 1}>I_{z z, 2} ; I_{T, 1}<I_{T, 2}$
$\square I_{y y, 1}=I_{y y, 2} ; I_{z z, 1}<I_{z z, 2} ; I_{T, 1}<I_{T, 2}$
$\square I_{y y, 1}>I_{y y, 2} ; I_{z z, 1}<I_{z z, 2} ; I_{T, 1}=I_{T, 2}$
$\square I_{y y, 1}=I_{y y, 2} ; I_{z z, 1}<I_{z z, 2} ; I_{T, 1}>I_{T, 2}$
$\square I_{y y, 1}>I_{y y, 2} ; I_{z z, 1}=I_{z z, 2} ; I_{T, 1}=I_{T, 2}$
e)*


$$
t_{3}>t_{2}>t_{1}
$$

Shown is the thin-walled cross section of a bar. The bar is loaded by a torsional moment. At which point would one expect the greatest magnitude of the torsional shear stress $|\tau|$ ?
$\square$ D
$\square \mathrm{C}$
$\square$ B
$\square \mathrm{A}$

## Problem 2 Plane truss (19 credits)

Given is the shown plane truss made of rigid bars. It is supported in the points $A$ and $B$. The system is loaded by point loads at points C, D, E, G, and H. Bearing conditions, loads and dimensions are shown in the drawing.

Given quantities: a, F, coordinate system xy (right-hand system)


All subtasks can be solved independently of each other.
a)* Is the truss statically determined?
$\square$ Yes
b)* How many degrees of freedom are constrained by the support in A?
$\square 2$
$\square 1$
$\square 0$
$\square 3$
c)* Calculate the support forces $A_{x}$. Consider the constant $k=2$ in point G .
$\square A_{x}=-4.5 \mathrm{~F}$
$\square A_{x}=7.0 \mathrm{~F}$$A_{x}=-9.0 F$
$\square A_{x}=3.5 \mathrm{~F}$
d)* Calculate the support forces $A_{y}$. Consider the constant $k=2$ in point $G$.
$\square A_{y}=-2.5 \mathrm{~F}$
$\square A_{y}=8.0 \mathrm{~F}$$A_{y}=-6.0 \mathrm{~F}$
$\square A_{y}=4.0 \mathrm{~F}$
e)* Calculate the support forces $B_{x}$. Consider the constant $k=2$ in point $G$.
$\square B_{x}=0 \mathrm{~F}$
$\square B_{x}=3.0 \mathrm{~F}$
$\square B_{x}=4.5 \mathrm{~F}$
$\square B_{x}=6.5 \mathrm{~F}$
f)* Calculate the bar force $S_{1}$ in bar 1 . Consider the constant $k=4$ in point $G$.

Given: The support forces are given with the values being $A_{x}=-\frac{15}{2} F, A_{y}=6 F$ and $B_{x}=\frac{15}{2} F$.

Convention: The positive direction of the support forces is aligned with the positive direction of the coordinate system axes.
$\square S_{1}=-7.0 \mathrm{~F}$
$\square S_{1}=-5.0 \mathrm{~F}$
$\square S_{1}=-6.0 \mathrm{~F}$
$\square s_{1}=-4.0 \mathrm{~F}$
g)* Calculate the bar force $S_{2}$ in bar 2. Consider the constant $k=4$ in point G .

Given: The support forces are given with the values being $A_{x}=-\frac{15}{2} F, A_{y}=6 F$ and $B_{x}=\frac{15}{2} F$.

Convention: The positive direction of the support forces is aligned with the positive direction of the coordinate system axes.
$\square S_{2} \approx 3.58 \mathrm{~F}$
$\square S_{2} \approx-9.41 \mathrm{~F}$
$\square S_{2} \approx-12.73 F$
$\square S_{2} \approx 7.21 \mathrm{~F}$
h)* Calculate the bar force $S_{3}$ in bar 3 . Consider the constant $k=4$ in point G .

Given: The support forces are given with the values being $A_{x}=-\frac{15}{2} F, A_{y}=6 F$ and $B_{x}=\frac{15}{2} F$.

Convention: The positive direction of the support forces is aligned with the positive direction of the coordinate system axes.
$\square S_{3}=3.0 \mathrm{~F}$
$\square S_{3}=-9.0 \mathrm{~F}$
$\square S_{3}=-5.0 \mathrm{~F}$
$\square S_{3}=4.0 \mathrm{~F}$

## Problem 3 ( 6 credits)

Given is the shown planar system. The massless, rigid bar (length $3 /$ ) is supported at points $A$ and $B$ and is support against the rigid environment at point B by a linear, mass-free spring (spring stiffness $k$ ). Two point masses $m$ and $m$ and are attached to the bar. The position of the bar is completely and unambiguously described by the angular coordinate $\phi$. The linear spring is relaxed in the position $\phi=\frac{\pi}{2}$. The acceleration due to gravity $g$ acts in the negative $y$-direction. Bearing conditions, loads as well as dimensions can be taken from the drawing.

Given quantities: $I, k, m, g$, coordinate systems xy (right-handed systems), Angular coordinate $\phi \in\left[0, \frac{p i}{2}\right]$


The system has two equilibrium positions. Cross the two correct equilibrium conditions.

D $\phi=\frac{2 \pi}{3}$
$\square \phi=\frac{\pi}{2}$
[ ${ }^{1}=\frac{\pi}{3}$

- $\phi=\arcsin \frac{3 m q}{2 \pi}$
$\square \phi=\frac{\pi}{4}$
$\square \phi=\arcsin \frac{m g}{4 k l}$
$\square \phi=\arcsin \frac{m g}{3 k l}$
$\square \phi=\arcsin \frac{2 m g}{3 k l}$


## Problem 4 Cross-Section Properties ( 8 credits)



Shown is the cross section of a beam with measurement a.
Given: a, ( $\bar{y}, \bar{z})$-coordinate-system
a)* Determine the coordinate $\overline{y_{c}}$ of the cross-section's centroid with respect to the given $\left(\bar{y}_{c}, \bar{z}_{c}\right)$-coordinatesystem.
$\square \bar{y}_{c}=2.41 \mathrm{a}$
$\square \overline{y_{c}}=2.52 \mathrm{a}$
$\square \bar{y}_{c}=2.19 a$
$\square \overline{y_{C}}=2.14 a$
b) ${ }^{*}$ Determine the coordinate $\bar{z}_{c}$ of the cross-section's centroid with respect to the given $\left(\bar{y}_{c}, \bar{z}_{c}\right)$-coordinatesystem.
$\square \bar{z}_{c}=2.52 a$
$\square \bar{z}_{c}=2.14 a$
$\square \bar{z}_{c}=2.19 a$
$\square \bar{z}_{c}=2.41 a$
c)* For this subtask, use the rounded centroid coordinates $\overline{y_{c}}=2.2 a$ and $\bar{z}_{c}=2.15 a$. Compute the second moment of area $I_{y y}$ with respect to the centroidal $(y, z)$-coordinate-system.
$\square l_{y y}=19.32 a^{4}$
$\square I_{y y}=17.71 \mathrm{a}^{4}$
$\square I_{y y}=18.71 a^{4}$
$\square I_{y y}=20.43 a^{4}$

## Problem 5 Beam Stresses (15 credits)

## Loadcase 1:



Loadcase 2:


The shown clamped beam of length $I$ is subjected to the moment $M$ about the $y$-axis and the normal force $F_{1}$ at it's free end in loadcase 1 and to the force $F_{2}$ in loadcase 2. The cross-section and the modulus of elasticity are constant along the beam's x-axis. Each subtask is referring to only one of both loadcases. Use the given coordinate system with its origin in the cross sections centroid $C$.
Given: $I_{z z}=8748 \mathrm{~mm}^{4},(x, y, z)$-coordinate-system, $a=18 \mathrm{~mm}, I=1 \mathrm{~m}, M=40 \mathrm{kNmm}, F_{1}=10 \mathrm{kN}, F_{2}=20$ kN
$(y, z)$-coordinates of points A, B, D and E: A: $\left(\frac{a}{2}, \frac{-a}{2}\right) ; \mathrm{B}:\left(\frac{a}{2}, \frac{a}{2}\right) ; \mathrm{D}:\left(\frac{-a}{4}, \frac{a}{2}\right)$; $\mathrm{E}:\left(\frac{-a}{4}, 0\right)$
a)* Loadcase 1: Calculate the normal stress $\sigma_{x x}$ at point A at $x=\frac{1}{2} I$
$\square \sigma_{x x, A}=72.03 \mathrm{MPa}$
$\square \sigma_{x x, A}=-72.03 \mathrm{MPa}$
$\square \sigma_{x x, A}=10.29 \mathrm{MPa}$
$\square \sigma_{x x, A}=-10.29 \mathrm{MPa}$
b)* Loadcase 1: Calculate the normal stress $\sigma_{x x}$ at point B at $x=\frac{1}{2} /$
$\square \sigma_{x x, B}=-10.29 \mathrm{MPa}$
$\square \sigma_{x x, B}=-72.03 \mathrm{MPa}$
$\square \sigma_{x x, B}=10.29 \mathrm{MPa}$
$\square \sigma_{x x, B}=72.03 \mathrm{MPa}$
c) Loadcase 1: Which statement is correct?
$\square$ Point A is stress free
$\square$ Point A is in compression
$\square$ Point A is in tension
d) Loadcase 1: Which statement is correct?
$\square$ Point B is in tension
$\square$ Point $B$ is stress free
$\square$ Point $B$ is in compression
e) Loadcase 1: Calculate $F_{1}$, such that the whole beam is in tension at $x=\frac{1}{2} /$.
$\square F_{1}>13.33 N$
$\square F_{1}<13.33 N$
$\square F_{1}>13.33 \mathrm{kN}$
$\square F_{1}<13.33 \mathrm{kN}$
f)*
g)* Loadcase 2: Calculate the shear stress $\tau_{x z}$ at point E at $x=\frac{1}{2} l$
$\square \tau_{x z, E}=0$
$\square \tau_{x z, E}=185.18 \mathrm{MPa}$
$\square \tau_{x z, E}=92.59 \mathrm{MPa}$
$\square \tau_{x z, E}=30.86 \mathrm{MPa}$

## Problem 6 Energy Methods (6 credits)



Given: $I=3 \mathrm{~m} ; q_{0}=150 \frac{\mathrm{~N}}{\mathrm{~m}} ; E=210 \mathrm{GPa}=210 * 10^{9} \frac{\mathrm{~N}}{\mathrm{~m}^{2}} ; I_{y y}=6 * 10^{-8} \mathrm{~m}^{4} ; G A_{s}=\infty$
a)* For the depicted system, the polynomial order of the bending moment $M(x)$ will be:

$\square$ Quadratic
b) * Compute $M(x=l)$.
$\square-112.5 \mathrm{Nm}$
$\square-225 \mathrm{Nm}$

Cubic
$\square$ Linear
$\square 112.5 \mathrm{Nm}$
$\square 225 \mathrm{Nm}$
c)* To calculate the deflection $w$ of the beam at $x=0$, one could use energy methods and apply a virtual force in the direction of the sought deflection. For the virtual "1"-System with the applied virtual force, the polynomial order of the bending moment $\bar{M}(x)$ would be:

| $\square$ Quadratic | $\square$ Linear |
| :--- | :--- |
| $\square$ Cubic | $\square$ Constant |

d) ${ }^{*}$ Compute $\bar{M}(x=l)$.
$\square-3 \mathrm{Nm}$
$\square 3 \mathrm{Nm}$
$\square 1.5 \mathrm{Nm}$
$\square-1.5 \mathrm{Nm}$
e)* Compute the deflection $w$ at $x=0$.
$\square w=0.0214 m$
$\square w=0.0401 m$
$\square w=0.0442 m$
$\square w=0.0321 \mathrm{~m}$

