## Qualifying examination master's course Aerospace

## Basics of thermodynamics

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## Example examination

Date of the exam Garching, June 12, 2020
Room MW 1250

Start of the exam 9:00 a.m.
Duration of the exam 60 minutes
Type of exam written
Approved tools Writing and drawing utensils, calculator, dictionary

## Exam-ID: <br> 1

Name, first Name:

## Student number:

The test information includes 15 Pages (including cover sheet). In addition, a formula collection is issued. Please check the completeness immediately after receiving the test information. Label this cover page with your last name, first name and your student number. Sign this cover sheet too. At the end of the test, all worksheets and the formula collection must be submitted.

All tasks are to be answered on the task sheets (not on the formula collection). Please use the front and back of the sheets. Calculation results must be entered in the fields provided and are only evaluated with solution path. If partial tasks cannot be solved, make reasonable assumptions for the missing values for further calculation.

## Signature:

## Achievable points

| Task: | 1 | 2 | 3 | 4 | 5 | Sum |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Points: | 10 | 8 | 9 | 11 | 32 | 70 |
| Achieved points: |  |  |  |  |  |  |

## 1. Definitions and terms

(a) When is a system in thermodynamic equilibrium?

## Solution:

A system is in thermodynamic equilibrium if its state variables do not change.
(b) Define the term phase!

## Solution:

A phase is an area of matter that can be delimited from the environment and has the same thermodynamic, physical and chemical properties.
(c) Characterize the following systems according to their properties:

$$
\begin{gathered}
\text { open - closed } \\
\text { diabat - adiabatic } \\
\text { static - dynamic } \\
\text { homogeneous - heterogeneous - continuous }
\end{gathered}
$$

If necessary, note the system limits shown!
i. Fluid in a perfectly insulated turbine

ii. Gas in a heat-permeable cylinder

iii. Air in the balloon with escaping gas


## Solution:

i. open/adiabatic/static/continuous
ii. closed/diabatic/static/homogeneous
iii. open/diabatic/static or dynamic/homogeneous

## 2. Thermal condition and ideal gas

(a) Name the three thermal state variables!!

## Solution:

Pressure, temperature, specific or molar volume.
(b) Name the thermal equation of state of an ideal gas as a function of the universal gas constant $R_{m}$ and the molar mass $M$.

## Solution:

$$
p \cdot v=\frac{R_{m}}{M} \cdot T
$$

(c) In a container with the volume $V=2 \mathrm{~m}^{3}$ is the mass $m=3 \mathrm{~kg}$ of an ideal gas ( $R_{m}=8.314 \mathrm{~J} / \mathrm{molK}$ ). The pressure is $p=1.5 \mathrm{bar}$, the temperature $T=350 \mathrm{~K}$
i. What is the molar mass $M$ and the amount of substance $n$ of the gas.
ii. The container is filled until its pressure is $p_{2}=2$ bar. How big is the new mass of the gas in the container? Assume that the filling process is isothermal.

## Solution:

i.

$$
\begin{gathered}
p \cdot V=m \cdot \frac{R_{m}}{M} \cdot T \\
M=m \cdot \frac{R_{m}}{p \cdot V} \cdot T=29.1 \mathrm{~kg} / \mathrm{kmol} \\
n=\frac{m}{M}=0.103 \mathrm{kmol}
\end{gathered}
$$

ii.

$$
m_{2}=\frac{p_{2} \cdot V \cdot M}{R_{m} \cdot T}=4 \mathrm{~kg}
$$

## 3. First law of thermodynamics

(a) Describe the statement of the 1st law of thermodynamics in your own words!

## Solution:

The energy of a system can be changed in the form of heat, work, as well as the inflow and outflow of enthalpy, kinetic energy and potential energy.
(b) Write down the 1st law of thermodynamics in general form as a power balance!

## Solution:

$$
\begin{array}{r}
\frac{d E_{\text {sys }}}{d t}=\sum_{i} \dot{Q}_{i}(t)+\sum_{j} \dot{W}_{j}(t)+ \\
\sum_{e} \dot{m}_{e}\left(h+\frac{c^{2}}{2}+g z\right)_{e}-\sum_{i} \dot{m}_{i}\left(h+\frac{c^{2}}{2}+g z\right)_{i}
\end{array}
$$

(c) In a steam boiler, $\dot{m}=10^{5} \mathrm{~kg} / \mathrm{h}$ water are evaporated every hour and the steam is expanded in a heat-insulated turbine. The specific enthalpy of the water at the boiler inlet is $h_{1}=128 \mathrm{~kJ} / \mathrm{kg}$, that of the steam at the boiler outlet and turbine inlet is $h_{2}=3150 \mathrm{~kJ} / \mathrm{kg}$ and at the turbine outlet $h_{3}=2350 \mathrm{~kJ} / \mathrm{kg}$.


Abbildung 1: Sketch of the system
i. How large is the heat flow supplied in the boiler $\dot{Q}_{K}$ ?
ii. What is the efficiency of the entire system?

## Solution:

i. System boiler (operated stationary):

$$
\begin{gathered}
\frac{d E_{\text {sys }}}{d t}=\sum_{i} \dot{Q}_{i}(t)+\sum_{j} \dot{W}_{j}(t)+\sum_{l} \dot{m}_{e}\left(h_{l}+\frac{c_{l}^{2}}{2}+g \cdot z_{l}\right) \\
0=\dot{Q}_{K}+\dot{m}\left(h_{1}-h_{2}\right) \\
\dot{Q}_{K}=-\dot{m}\left(h_{1}-h_{2}\right)=83.9 \mathrm{MW}
\end{gathered}
$$

ii. System turbine (operated stationary):

$$
\begin{gathered}
\frac{d E_{\text {sys }}}{d t}=\sum_{i} \dot{Q}_{i}(t)+\sum_{j} \dot{W}_{j}(t)+\sum_{l} \dot{m}_{e}\left(h_{l}+\frac{c_{l}^{2}}{2}+g \cdot z_{l}\right) \\
0=\dot{W}_{T}+\dot{m}\left(h_{2}-h_{3}\right) \\
\dot{W}_{T}=P_{T}=-22.2 \mathrm{MW}
\end{gathered}
$$

This results in the efficiency:

$$
\eta_{g e s}=\frac{\left|P_{T}\right|}{\dot{Q}_{K}}=0.265
$$

4. Change of state and second law of thermodynamics
(a) Air compression: 10 kg air (ideal gas, $\kappa=1.4, R_{\text {Luft }}=287 \mathrm{~J} / \mathrm{kgK}$ ) are compressed from $T_{1}=293 \mathrm{~K}$ and $p_{1}=0.95$ bar to $p_{2}=10 \mathrm{bar}$. This is supposed to happen
5. isochorously
6. isothermal
7. reversibly adiabatically and
8. polytropically with $n=1.3$.
i. Sketch the respective state change in the $p-v$ diagram!
ii. Calculate the final temperature $T_{2}$, the final volume $V_{2}$ and the transferred heat $Q$ for the reversible adiabatic case!

## Solution:


i.
ii.

$$
\begin{gathered}
\frac{p_{2}}{p_{1}}=\left(\frac{T_{2}}{T_{1}}\right)^{\frac{\kappa}{\kappa-1}} \\
T_{2}=T_{1} \cdot\left(\frac{p_{2}}{p_{1}}\right)^{\frac{\kappa-1}{\kappa}}=572.33 \mathrm{~K}
\end{gathered}
$$

$$
V_{2}=\frac{m \cdot R \cdot T_{2}}{p_{2}}=1.64 \mathrm{~m}^{3}
$$

For the heat $Q=0$ applies because it is an adiabatic change of state..
(b) Check whether the following statements about entropy are right or wrong. Incorrect answers result in point loss within the task. The minimum score within a task is 0.
i. The entropy $S$ is a state variable.
ii. The entropy $S$ is a measure of the likelihood that a condition will occur.
iii. The entropy of a system can never decrease.
iv. Entropy can also be generated in reversible systems.
v. Systems can exchange entropy with the environment by transferring heat across the system boundary.

## Solution:

i. right
ii. right
iii. wrong
iv. wrong
v. right

## 5. Steam power plant

A large coal-fired power plant works in a simplified manner according to the following process:

- Saturated water in the feed water tank is at a pressure of $p_{1}=0.1 \mathrm{MPa}$ (state 1 ).
- The liquid water is brought to pressure $p_{2}=7 \mathrm{MPa}$ by the isentropic feed water pump ( $1 \rightarrow 2$ ).
- In the boiler, the water is preheated to the boiling line by the heat emitted when the coal is burned $(2 \rightarrow 3)$, completely evaporated $(3 \rightarrow 4)$ and superheated $(4 \rightarrow$ 5). A heat flow of $\dot{Q}_{t h, 2-5}=1.3 \mathrm{GW}$ is transferred from the boiler to the working medium. While preheating and evaporation are isobaric, there is a pressure loss of $\Delta p_{4-5}=0.5 \mathrm{MPa}$ while overheating.
- In the adiabatic high-pressure turbine $(5 \rightarrow 6)$ the water is expanded to a pressure of $p_{6}=1 \mathrm{MPa}$ polytropic. The isentropic efficiency of the high pressure turbine is $\eta_{\text {is }}=0.8$
- In the isobaric reheater $(6 \rightarrow 7)$ the water is reheated up to temperature $T_{7}=T_{5}$.
- In the adiabatic low-pressure turbine (7 $\rightarrow 8$ ), the water is finally released up to pressure $p_{8}=5 \mathrm{kPa}$. There is wet steam in state 8 . Losses result in an entropy flow of $\dot{S}_{i r r, 7-8}=51 \frac{\mathrm{~kW}}{\mathrm{~K}}$ in the low-pressure turbine.
- The wet steam is fully condensed in the condenser ( $8 \rightarrow 9$ ). The heat generated is removed by cooling water, which is taken from the neighboring river.
- With the help of an auxiliary pump and the use of waste heat in the power plant, the water is finally brought back to the state of the tank (state 1).



## Abbildung 2: Sketch of the power plant process

## Given Values:

Mass flow of the process: $\dot{m}=420 \mathrm{~kg} / \mathrm{s}$
Storage: $p_{1}=0.1 \mathrm{MPa}, v_{1}=0,001 \mathrm{~m}^{3} / \mathrm{kg}$
Feed water pump: $p_{2}=7 \mathrm{MPa}$
Main boiler: $\dot{Q}_{t h, 2-5}=1.3 \mathrm{GW}, \Delta p_{4-5}=0.5 \mathrm{MPa}$
High pressure turbine: $p_{6}=1 \mathrm{MPa}, \eta_{i s}=0,8,\left.\bar{c}_{p}\right|_{5} ^{6}=1909.25 \frac{\mathrm{~J}}{\mathrm{~kg} \cdot \mathrm{~K}}, \kappa=1.28$
Low pressure turbine: $p_{8}=5 \mathrm{kPa}, \dot{S}_{i r r, 7-8}=51 \frac{\mathrm{~kW}}{\mathrm{~K}}$

## Hints:

- All pressures are absolute pressures.
- Liquid water can be taken as an incompressible liquid, superheated steam as an ideal gas.
- Flow velocities and height differences can be neglected in all subtasks.
- If intermediate values from the steam tables are required, interpolate linearly between two neighboring values. Extrapolation of the values is not permitted.


## Steam tables:

Saturation state:

| $p$ <br> MPa | $T$ <br> ${ }^{\circ} \mathrm{C}$ | $h^{\prime}$ <br> $\frac{\mathrm{kJ}}{\mathrm{kg}}$ | $h^{\prime \prime}$ <br> $\frac{\mathrm{kJ}}{\mathrm{kg}}$ | $s^{\prime}$ <br> $\frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}$ | $s^{\prime \prime}$ <br> $\frac{\mathrm{kJ}}{\mathrm{kg} \cdot \mathrm{K}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.005 | 32.88 | 137.72 | 2560.5 | 0.476 | 8.393 |
| 0.1 | 99.63 | 417.51 | 2675.1 | 1.303 | 7.359 |
| 7 | 285.86 | 1267.9 | 2771.8 | 3.121 | 5.813 |

Superheated steam:

| $p$ <br> MPa | $T$ <br> ${ }^{\circ} \mathrm{C}$ | $h$ <br> $\frac{\mathrm{~kJ}}{\mathrm{~kg}}$ | $s$ <br> $\frac{\mathrm{~kJ}}{\mathrm{~kg} \cdot \mathrm{~K}}$ |
| :---: | :---: | :---: | :---: |
| 1 | 450 | 3370.8 | 7.6190 |
| 1 | 500 | 3478.3 | 7.7627 |
| 1 | 550 | 3587.1 | 7.8991 |
| 1 | 600 | 3697.4 | 8.0292 |
| 6.5 | 450 | 3296.3 | 6.680 |
| 6.5 | 500 | 3416.4 | 6.841 |
| 6.5 | 550 | 3534.6 | 6.989 |
| 6.5 | 600 | 3652.1 | 7.127 |
| 7 | 450 | 3289.1 | 6.637 |
| 7 | 500 | 3410.6 | 6.799 |
| 7 | 550 | 3529.6 | 6.948 |
| 7 | 600 | 3647.9 | 7.088 |

Tasks:
(a)

Qualitatively draw the T-s diagram of the power plant process. Draw in the following sizes and label them:

- Boiling line (BL), Dew line (DL) and critical point (CP). Label the areas „supercooled liquid " (cl), „Two-phase region " (2PR) and „superheated steam" (hs).
- States 1-9 of the process as points.
- The state changes between the points. Pay attention to the characteristic course of the respective state change.
- The isobaric line for $p=p_{2}$. The line must go through all three areas (supercooled liquid, wet steam area, superheated steam).

(b)

Determine the temperature $T_{1}$ and the specific enthalpy $h_{1}$ in the feed water tank.

## Solution:

From the steam table for the state of saturation:

$$
\begin{aligned}
& T_{1}=99.63^{\circ} \mathrm{C} \\
& h_{1}=h^{\prime}=417.51 \frac{\mathrm{~kJ}}{\mathrm{~kg}}
\end{aligned}
$$

(c)

Calculate the power delivered from the feed water pump to the working medium $P_{F W P}$. Also specify the temperature $T_{2}$ at the pump outlet.

## Solution:

Compression work for an incompressible liquid:

$$
\begin{aligned}
P_{F W P} & =\dot{W}_{T, F W P}=\dot{m} \cdot w_{T, F W P}= \\
& =\dot{m} \cdot v_{1} \cdot\left(p_{2}-p_{1}\right)=2.898 \mathrm{MW} \\
T_{2} & =T_{1}=99.63^{\circ} \mathrm{C}
\end{aligned}
$$

(d)

Calculate the temperature $T_{5}$ at the inlet of the high pressure turbine.

## Solution:

First law of thermodynamics (power balance):

$$
\frac{d E_{s y s}}{d t}=\dot{m}_{2} \cdot h_{2}-\dot{m}_{5} \cdot h_{5}+\dot{Q}_{t h, 2-5}=0
$$

With:

$$
\begin{aligned}
\dot{m}_{2} & =\dot{m}_{5}=\dot{m} \\
h_{2} & =h_{1}+v_{1} \cdot\left(p_{2}-p_{1}\right)=424.41 \frac{\mathrm{~kJ}}{\mathrm{~kg}}
\end{aligned}
$$

For: $h_{5}$ :

$$
h_{5}=h_{2}+\frac{\dot{Q}_{t h, 2-5}}{\dot{m}}=3519.65 \frac{\mathrm{~kJ}}{\mathrm{~kg}}
$$

Calculation of the pressure $p_{5}$ :

$$
p_{5}=p_{2}-\Delta p_{4-5}=6.5 \mathrm{MPa}
$$

Using the steam table results from linear interpolation:

$$
T_{5}=543.68^{\circ} \mathrm{C}=816.83 \mathrm{~K}
$$

(e)

Calculate the temperature $T_{6}$ at the outlet of the high pressure turbine and the amount of power $\left|P_{H P T}\right|$ delivered by the high pressure turbine.

## Solution:

Definition of the isentropic efficiency of a turbine with ideal gas as the working medium:

$$
\eta_{i s, T}=\frac{\left|w_{t}\right|}{\left|w_{t, i s}\right|}=\frac{h_{a}-h_{e}}{h_{a, i s}-h_{e}}=\frac{T_{a}-T_{e}}{T_{a, i s}-T_{e}}
$$

With $T_{e}=T_{5}$ and $T_{a}=T_{6}$.
Isentropic change of state:

$$
\begin{aligned}
& \frac{T_{6, i s}}{T_{5}}=\left(\frac{p_{6}}{p_{5}}\right)^{\frac{\kappa-1}{\kappa}} \\
& T_{6, i s}=T_{5} \cdot\left(\frac{p_{6}}{p_{5}}\right)^{\frac{\kappa-1}{\kappa}}=542.38 \mathrm{~K}
\end{aligned}
$$

It results in:

$$
\begin{aligned}
T_{6} & =T_{5}+\eta_{i s, T} \cdot\left(T_{6, i s}-T_{5}\right)=597.27 \mathrm{~K} \\
\left|P_{H P T}\right| & =\left|\dot{m} \cdot \bar{c}_{p}\right|_{5}^{6} \cdot\left(T_{6}-T_{5}\right) \mid=176.06 \mathrm{MW}
\end{aligned}
$$

(f)

How big is the steam content $x_{8}$ (respectively the wetness $\left(1-x_{8}\right)$ at the outlet of the low pressure turbine? Why is excessive wetness in the low pressure turbine problematic?

## Solution:

State variables at the inlet of the turbine:

$$
\begin{aligned}
T_{7} & =T_{5}=816.83 \mathrm{~K} \\
p_{7} & =p_{6}=1 \mathrm{MPa} \\
s_{7} & =7.8819 \frac{\mathrm{~kJ}}{\mathrm{~kg} \cdot \mathrm{~K}}
\end{aligned}
$$

Second law of thermodynamics (entropy balance):

$$
\frac{d S_{s y s}}{d t}=\dot{m}_{7} \cdot s_{7}-\dot{m}_{8} \cdot s_{8}+\dot{S}_{i r r, 7-8}=0
$$

With:

$$
\dot{m}_{7}=\dot{m}_{8}=\dot{m}
$$

Finally, for $s_{8}$ :

$$
s_{8}=s_{7}+\frac{\dot{S}_{i r r, 7-8}}{\dot{m}}=8.0033 \frac{\mathrm{~kJ}}{\mathrm{~kg} \cdot \mathrm{~K}}
$$

For each state variable $z$ in the 2-phase region:

$$
z=(1-x) \cdot z^{\prime}+x \cdot z^{\prime \prime}=z^{\prime}+x\left(z^{\prime \prime}-z^{\prime}\right)
$$

This results in the steam content respectively the wetness::

$$
\begin{aligned}
x_{8} & =\frac{s_{8}-s^{\prime}}{s^{\prime \prime}-s^{\prime}}=0.95 \\
\left(1-x_{8}\right) & =0.05
\end{aligned}
$$

The values for $s^{\prime}$ and $s^{\prime \prime}$ are taken from the steam table for $p=p_{8}$.
Excessive wetness in the low-pressure turbine leads to erosion of the turbine blades and must therefore be avoided.

