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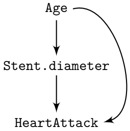
Nonparametric Effect Estimation and Guideline Search for Continuous Treatments

Motivating Example

- We want to measure the effect of the stent diameter on the heart attack rate. Is `lm(HeartAttack ~ Stent.diameter)` an appropriate model?

- How can we estimate the effect non-parametrically assuming `Stent.diameter` is continuous?

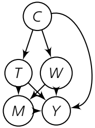


- And what is a good guideline telling physicians how to select the stent diameter based on the patient's age?

Do-Operator and Interventions [Pearl, 2009]

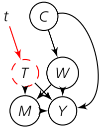
We are assuming a continuous SEM $X_i = h_i(X_{\text{pa}(i)}, \varepsilon_i)$, ε_i independent, with a binary response $Y = h_Y(X_{\text{pa}(Y)}, \varepsilon_Y)$. No unobserved confounding.

P_X



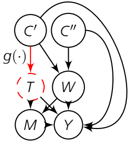
$$\begin{aligned} C &= \varepsilon_1 \\ T &= h_1(C, \varepsilon_2) \\ W &= h_2(C, \varepsilon_3) \\ M &= h_3(T, W, \varepsilon_4) \\ Y &= h_4(C, M, W, \varepsilon_5) \end{aligned}$$

$P_{X;do(T=t)}$



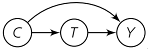
$$\begin{aligned} C &= \varepsilon_1 \\ T &= \color{red}{t} \\ W &= h_2(C, \varepsilon_3) \\ M &= h_3(T, W, \varepsilon_4) \\ Y &= h_4(C, M, W, \varepsilon_5) \end{aligned}$$

$P_{X;do(T=g(A))}$



$$\begin{aligned} (C', C'') &= \varepsilon_1 \\ T &= \color{red}{g(C')} \\ W &= h_2(C, \varepsilon_3) \\ M &= h_3(T, W, \varepsilon_4) \\ Y &= h_4(C, M, W, \varepsilon_5) \end{aligned}$$

Marginal Integration (MI, Backdoor Adjustment)

Under 

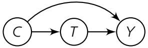
, the average causal effect can be written as an observational quantity:

$$\mathbb{E}[Y; do(T = g(C))] = \int \mathbb{E}[Y|c, T = g(C)] f(c)\, \mathrm{d}c \approx \frac{1}{n} \sum_{i=1}^{n} \hat{\mathbb{E}}[Y|C^i, T = g(C^i)]$$

In practice, this requires estimating the conditional expectation (linear model, logistic regression, local linear estimator) and approximating the integral.

[Ernest and Bühlmann, 2015]

Inverse Probability Weighting

Again, 

. For this slide, assume T binary:

$$\mathbb{E}[Y; do(T = g(C))] = \int yf(y|a, T = g(c))f(c)\,\mathrm{d}(y, c) = \mathbb{E}\left[Y \frac{\mathbb{1}_{T=g(C)}}{f(T|C)}\right]$$

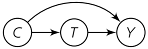
Example

	T	f(T|C)
Alice	1	0.2
Bob	0	0.4
Charlie	1	0.5

Goal: $\mathbb{E}[Y; do(T = 1)]$
Virtual dataset: $\tilde{D} := \{A, A, A, A, A, C, C\}$
Estimate interventional expectation as average of Y in $\tilde{D}$.

[Robins et al., 1994]

Continuous IPW

Assume 

. Let T be continuous and K^n be kernels.

$$\mathbb{E}[Y; do(T = g(C))] \overset{n\to\infty}{\longleftarrow} \mathbb{E}\left[Y \frac{K^n(T - g(C))}{f(T|C)}\right] \approx \frac{1}{n}\sum_{i=1}^{n} Y^i \frac{K(T^i - C^i)}{\hat{f}(T^i|C^i)}$$

- $K(T - g(C))$ quantifies the distance of the natural treatment to the intervention guideline instead of $\mathbb{1}_{T=g(C)}$.
- In CIPW, the propensity score $f(T|C)$ has to be estimated opposed to the conditional expectation in MI.

[Kallus and Zhou, 2018]

Heuristic for (explainable) guidelines

Goal: Find guideline $\hat{g}(\cdot)$ with small $\mathbb{E}[Y; do(T = \hat{g}(C))]$

Idea: We have for any guideline g the natural lower bound

$$\mathbb{E}[Y; do(T = g(C))] \overset{MI}{\approx} \frac{1}{n} \sum \hat{\mathbb{E}}[Y|T = g(C^i), C^i] \geq \frac{1}{n} \sum \min_{\tau} \hat{\mathbb{E}}[Y|T = \tau, C^i].$$

Steps:

1. Find $\hat{\tau}^i = \text{argmin}_{\tau \in supp(T)} \hat{\mathbb{E}}[Y|T = \tau, C^i]$, the risk minimizing treatment for sample i.
2. Find an (explainable) guideline $\hat{g}$ such that $\hat{g}(C^i) \approx \hat{\tau}^i$ for all samples.
3. Estimate $\mathbb{E}[Y; do(T = \hat{g}(C))]$ with CIPW, which is based on $\hat{f}(C|T)$.

Advantage: Data-driven heuristic and not biased by expert assessment.

Dataset: Stent Implantation

Sets of Variables[1]**:**

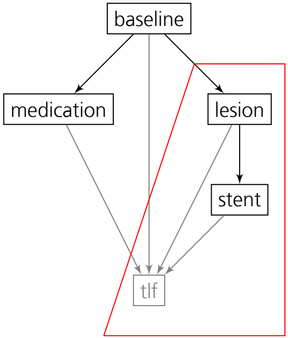
- baseline information of the patient (age, smoking status, nr. of prior heart attacks),
- medication information (anti-platelet, heparin),
- lesion information of the target vessel (length, diameter, residual blood flow, eccentricity),
- stent information (length, diameter, applied pressure, inflation time) and
- target lesion failure [response, binary] (heart attack or revascularization) information for up to 5 years.

Goal: Estimate intervention expectations $\mathbb{E}\left[\texttt{tlf}; do\left(\texttt{stent.attribute} = \xi\right)\right]$ and $\mathbb{E}\left[\texttt{tlf}; do\left(\texttt{stent.attribute} = \xi \cdot \texttt{lesion.attribute}\right)\right]$ and apply heuristic.

[1] In this talk, we neglect pre- and post-dilatation balloon information

Expert Knowledge Causal DAG

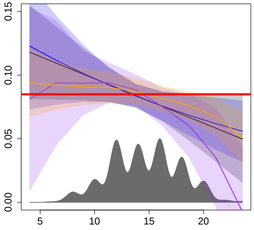
- We have baseline, medication $\perp\!\!\!\perp$ stent | lesion. Thus, we can restrict our analysis on the three red groups of random variables on the right.
- Every node may contain a random vector. To allow for slimmer regression models, we trim superfluous edges with based on conditional independence tests (assuming additive noise).



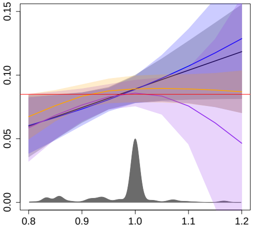
[Peters et al., 2014], similar to their phase 2

Results: Effect estimation

$\mathbb{E}[tlf; do(T = t)]$ against t



$\mathbb{E}[tlf; do(T = \xi \cdot C)]$ against ξ

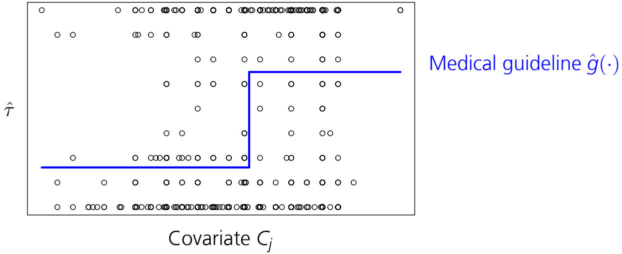


MI with linear model, MI with logistic regression, MI with local linear estimator, CIPW with kernel density estimator
dataset tlf risk, gray area: underlying density, shaded areas: 90% bootstrap CI

$\Rightarrow$ Recommend higher values of t | $\Rightarrow$ Recommend lower values of ξ

- Not shown here: similar results when computed with the original and trimmed DAG.

Results: Heuristic guideline



- Here, $\hat{g}(\cdot)$ is just a function of C_j. Simultaneously construct guidelines for all covariates and check intervention expectation.
- Few guidelines only seem to decrease the heart attack rate as often $\hat{g}(C_j) - \hat{\tau}$ is large.
- Explainable guidelines that can be directly implemented in hospitals and observed be further in future medical studies.

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