Interval decomposition and coarse graining

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In this talk I will present some results of my master‘s thesis. The first part is about interval decomposition. This part deals with a subcategory of persistence modules (with $(\mathbb{R}, <_{\mathbb{R}})$ as poset), which satisfy the following conditions. They are:

- Point-wise finite dimensional
- Semi-continuous
- and they admit admit a finite spectral set. (The spectral set of a persistence module is defined as the set of points at which a vector dies or is born, which we will define explicitly.)

Persistence modules induced by a Morse function on a closed manifold are an example of persistence modules that satisfy these conditions. Once we have allowed ourselves these restrictions, we can write the vector spaces at each point in a non-functorially decomposed form (in a non-canonical way), by which the information about the persistence of vectors at each point is reflected in the decomposition. The barcode of the persistence module can be determined by the dimension of some of these vector spaces.
In the second part I introduce a methode of coarse graining of barcodes. For a given coarsening parameter δ the barcode and its δ-coarsed barcode are δ-matched. Depending on the problem, the importance of capturing the "geometric similarity" by bottleneck distance may vary. In the figure below two S^2 manifolds are depicted. If you want to capture the geometric information about the similarity of these two objects and the "scale" is not a geometrically important information, the bottleneck distance will not reflect this similarity. So we may need to develop a descaled model of the barcode and this can be done by using the coarsening method. This starts by defining a descaling functor from $(\mathbb{R}^{\geq \mathbf{0}}, <_{\mathbb{R}})$ to the category of barcodes. Here the information about "scale" of the persistence module has not disappeared, but is stored differently.