Title: Applications of Topological Data Analysis in Mesoscale Eddy tracking

Abstract: Motivated by recent advances in climate science[1], we want to improve current models in order to be more well-equipped to make policy decisions to face the challenges of climate change. Current models do not generally resolve the mesoscale[2], and improvement of these centres around implementing gradually finer mechanisms. Therefore, we would like to better understand the role of mesoscale eddies in ocean heat and nutrient transport. This is done by using Topological Data Analysis (TDA) to study satellite data of Sea Level Anomalies (SLA) from the European Union's Copernicus Europe's eyes on Earth programme, exploiting the fact that we can approximately recover water velocity from the geostrophic balance[3]. The idea of tracking cyclonic and anticyclonic features has been implemented prior, and we will be building upon the modular framework developed by Engelke et al[4]. Specifically, after briefly introducing necessary notions from oceanography and topology, we will be focusing on developing a notion of index such that a theorem reminiscent of Poincare-Hopf[5] holds. We then go on to prove this result and show the equivalence of another definition of index with the potential of being generalised to higher dimensions. Finally, we discuss how we can pass between the index of critical points as we have defined them and nodes of the Reeb graph, and then go on to show some examples on data to see how this plays out in practice.

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[1] H-O Poertner et al. *Climate Change 2022: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change.* Cambridge University Press, 2022

[2] George Haller and Ryan Abernathy. *Transport by Lagrangian Vortices in the Eastern Pacific.* Journal of Physical Oceanography, 48:667-685, American Meteorological Society, 2018

[3] Talley et al. *Descriptive Physical Oceanography: An Introduction.* Elsevier, 2011

[4] Engelke et al. *Topology-based design and tracking for multi-center cyclones.* Topological Methods in Data Analysis and Visualization VI, pages 71-85, Springer, 2021

[5] Alan Pollack and Victor Guillemin. *Differential Topology.* AMS Chelsea Publishing, 1974