Sample Test: Aptitude Test "Elektrotechnik" - MRBE

Instructions

Answer all questions.

Part A: Multiple Choice (2 points each)

1. **Problem.** Which law states that the algebraic sum of currents at any node of an electric circuit is zero?

   (a) Ohm's Law

   (b) Kirchhoff's Voltage Law

   (c) Kirchhoff's Current Law

   (d) Tellegen's Theorem

   Answer: (c) Kirchhoff's Current Law (KCL). states that $\sum I_{\text{in}} = \sum I_{\text{out}}$, i.e., the net current into a node of an electric circuit is zero.

2. **Problem.** In a DC circuit, the maximum power is transferred to a resistive load when:

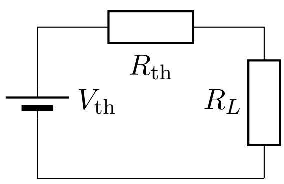
   (a) The load resistance is null.

   (b) The load resistance equals the source Thévenin equivalent resistance.

   (c) The load resistance is infinite.

   (d) The source voltage equals the load voltage.

   Answer: (b) For a linear source with Thévenin resistance R_{th}, the maximum power transferred to the load occurs when $R_L = R_{\text{th}}$. See proof below.



   i. The current through the load is

$$I = \frac{V_{\text{th}}}{R_{\text{th}} + R_L}.$$

   ii. The corresponding power transferred to the load is

$$P_L = I^2 R_L = \left(\frac{V_{\text{th}}}{R_{\text{th}} + R_L}\right)^2 R_L = \frac{V_{\text{th}}^2 R_L}{(R_{\text{th}} + R_L)^2}.$$

iii. Now we need to maximize P_L with respect to R_L, i.e., differentiate its function.

$$f(R_L) = \frac{V_{\text{th}}^2 R_L}{(R_{\text{th}} + R_L)^2}.$$

The first derivative is

$$\frac{df}{dR_L} = V_{\text{th}}^2 \cdot \frac{(R_{\text{th}} + R_L)^2 - 2R_L(R_{\text{th}} + R_L)}{(R_{\text{th}} + R_L)^4},$$

and, when we simplify its denominator, we find

$$(R_{\text{th}} + R_L)^2 - 2R_L(R_{\text{th}} + R_L) = (R_{\text{th}} + R_L)(R_{\text{th}} - R_L).$$

Thus,

$$\frac{df}{dR_L} \propto (R_{\text{th}} - R_L).$$

iv. Now we can set the derivative to zero, finding

$$R_{\text{th}} - R_L = 0 \implies R_L = R_{\text{th}}.$$

v. A second derivative test will tell us if the power is positive or negative. We will find that it is negative at $R_L = R_{\text{th}}$, so the power is maximum.

3. **Problem.** Which of the following is *not* a correct assumption for an ideal operational amplifier?

   (a) Its input impedance is infinite.

   (b) It has a zero output impedance.

   (c) Its bandwidth is infinite.

   (d) It has a finite open-loop gain.

   Answer: (d) An ideal op-amp presents an *infinite* open-loop gain (not *finite*), infinite input impedance, zero output impedance, and infinite bandwidth.

4. **Problem.** The power factor of a linear purely capacitive circuit is:

   (a) 1.

   (b) 1/2.

   (c) Leading.

   (d) Lagging.

   Answer: (c) Leading. For a pure capacitor, its current leads its terminal voltage by 90°. The magnitude of $\cos\varphi$, i.e. its power factor, is 0, and the sign (lead/lag) is *leading.*

5. **Problem.** Which of the following implements a *first-order low-pass* filter response? Assume the output is taken across the indicated element.

   (a) Series RL, output across L.

   (b) Series RC, output across C.

   (c) Parallel LC in a lossless circuit.

(d) Series RC, output across R.

Answer: (b) A series RC with the output across the capacitor is a first-order low-pass filter. We can think that at low frequency the capacitor impedance is high, so that higher voltages appear across C, while at high frequency the capacitor impedance is low and its voltage drops.

Part B: Short Answer (3 points each)

6. **Problem.** Explain the difference between *active*, *reactive*, and *apparent* power in AC linear circuits.

   Answer: Active power P (watts) is the real power dissipated or converted to work/heat. In a single-phase circuit, $P = VI\cos\varphi$, where V is the RMS value of the voltage, I is the RMS value of the current, and $\cos\varphi$ is the angle representing the displacement between the sinusoidal voltage and current. Reactive power Q (vars) is the power that oscillates between the circuit elements. In a single-phase circuit, $Q = VI\sin\varphi$. Apparent power S (VA) is the product $S = VI$ and satisfies $S^2 = P^2 + Q^2$; the power factor is $PF = \cos\varphi = P/S$.

7. **Problem.** State Thévenin's theorem and describe its practical application.

   Answer: Thévenin's theorem: Any linear, bilateral two-terminal network can be replaced by an equivalent voltage source V_{th} in series with a resistance R_{th} as seen from the terminals. **Application:** This theorem simplifies load analysis and design, e.g., selecting R_L for maximum power transfer. Example: Compute V_{th} as the open-circuit terminal voltage; compute R_{th} by turning off independent sources (i.e., voltage sources are shorted, and current sources are open circuits) and finding the equivalent resistance.

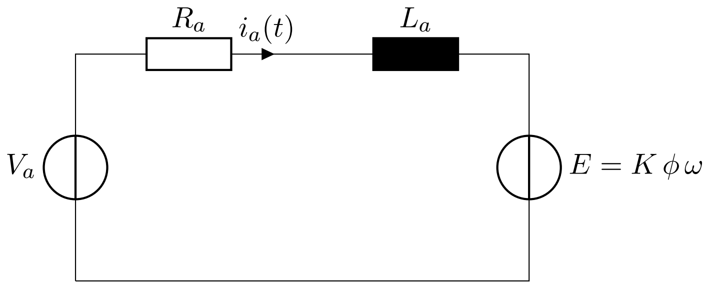
8. **Problem.** Draw a simplified equivalent circuit of a DC machine without its excitation windings and label its main components. Afterwards, calculate the average power at the machine shaft, neglecting any mechanical losses and assuming steady state conditions.

   Answer: A simple armature circuit of a separately excited field machine includes the source voltage V_a, the machine series armature resistance R_a, its inductance L_a, and its back-EMF source $E = K\phi\omega$ opposing the applied voltage. The electrical dynamics are then defined by the differential equation

$$V_a = E + i_a R_a + L_a \frac{di_a}{dt}, \qquad E = K\phi\,\omega,$$

   and the electromechanical torque is $T = K\phi\, i_a$.

DC machine armature equivalent



The average power developed at the machine shaft is given by

$$P = E \times i_a,$$

where

$$i_a = \frac{V_a - E}{R_a}.$$

Thus,

$$P = \frac{V_a\,E - E^2}{R_a}.$$

9. **Problem.** Define the quality factor Q in a linear RLC series circuit and explain its significance to analog filter design.

   Answer: For a series RLC at resonance ω_0,

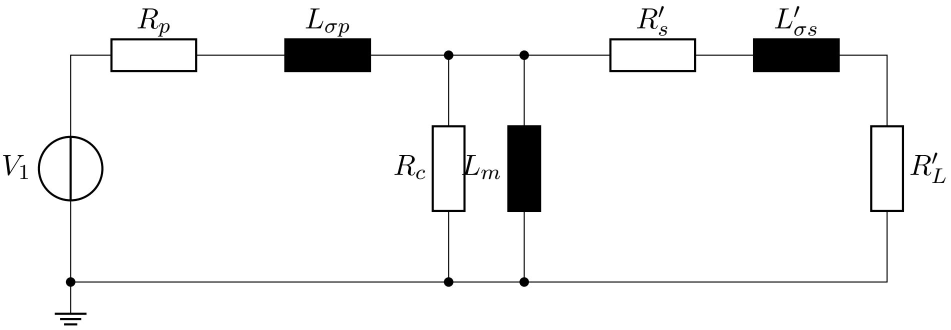
   $$Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

   Higher Q implies a sharper resonance, i.e., a narrower filter bandwidth $B = \omega_0/Q$, higher selectivity, and larger peak reactive currents or voltages for a given source.

10. **Problem.** Draw an equivalent electrical circuit of a two-winding transformer reflected to its primary side, depicting the magnetizing inductance, the leakage inductances, core losses, and winding losses. What is the role of *magnetizing inductance* in a transformer?

    Answer:

    Transformer equivalent referred to the primary

    The magnetizing inductance L_m models the current needed to establish the mutual magnetic flux in the core. It primarily determines the no-load current and influences voltage regulation and core excitation. In the equivalent circuit, L_m appears as a shunt branch (usually on the primary side) in parallel with core-loss resistance.

Part C: Problem Solving (4 points each)

11. **Problem.** A copper wire is 15 m long and has a diameter 1.5 mm. Compute its total electric resistance using a resistivity of $\rho_{\mathrm{Cu}} = 1.68 \times 10^{-8}\,\Omega\,\mathrm{m}$.

    Answer: The wire radius is $r = 0.75\,\mathrm{mm} = 7.5 \times 10^{-4}\,\mathrm{m}$ and its cross-sectional area

    $$A = \pi r^2 = \pi(7.5 \times 10^{-4})^2 = 1.767 \times 10^{-6}\,\mathrm{m}^2.$$

    Thus, the resistance is found with

    $$R = \rho\frac{L}{A} = \frac{1.68 \times 10^{-8} \times 15}{1.767 \times 10^{-6}} \approx 0.143\,\Omega.$$

12. **Problem.** For a series RLC with $L = 0.1\,\mathrm{H}$ and $C = 100\,\mu\mathrm{F}$, compute the resonant frequency f_0 of the circuit.

    Answer:

    $$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 100 \times 10^{-6}}} = \frac{1}{2\pi\sqrt{10^{-5}}} \approx 50.3\,\mathrm{Hz}.$$

13. **Problem.** A non-inverting amplifier uses $R_1 = 10\,\mathrm{k}\Omega$ connected to ground, and $R_2 = 30\,\mathrm{k}\Omega$ as a feedback resistor. What is the voltage gain of this amplifier?

    Answer: For a non-inverting amplifier:

$$A_v = 1 + \frac{R_2}{R_1} = 1 + \frac{30\,\mathrm{k}\Omega}{10\,\mathrm{k}\Omega} = 4.$$

14. **Problem.** A transformer has a primary side voltage withe an RMS value of 400 V. Its turns ratio is $N_p : N_s = 8 : 1$. Find the secondary side RMS voltage value.

    Answer:

$$V_s = V_p \frac{N_s}{N_p} = 400\,\mathrm{V} \times \frac{1}{8} = 50\,\mathrm{V}.$$

15. **Problem.** A DC machine draws 10 A at 220 V and delivers 2 kW mechanical output. Calculate its efficiency.

    Answer: Its input power is given by

$$P_{\mathrm{in}} = VI = 220 \times 10 = 2200\,\mathrm{W}.$$

The output power is $P_{\mathrm{out}} = 2000\,\mathrm{W}$. Thus, the efficiency of this machine is

$$\eta = \frac{P_{\mathrm{out}}}{P_{\mathrm{in}}} \times 100\% = \frac{2000}{2200} \times 100\% \approx \boxed{90.9\%}.$$